

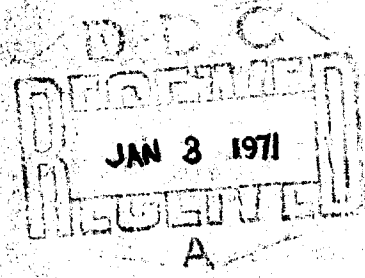
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SOURCE LEVELS OF SHALLOW UNDERWATER  
EXPLOSIONS

By  
Joel B. Gaspin  
Verna K. Shuler

13 October 1971



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NATIONAL ORDNANCE LABORATORY, WHITE OAK, SILVER SPRING, MARYLAND

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| All discrepancies have not and cannot be resolved without further comparisons with high-quality broadband measurements of shallow underwater explosions. It is tentatively recommended that shallow source levels, where the surface reflection cannot be separated out, be determined by the method of the present study.  |   |                                    |  |

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| underwater explosions as acoustic sources |        |    |        |    |        |    |
| depth variation                           |        |    |        |    |        |    |
| surface reflection                        |        |    |        |    |        |    |

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Prepared by:  
Joel B. Gaspin and Verna K. Shuler

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All discrepancies have not and cannot be resolved without further comparisons with high-quality broadband measurements of shallow underwater explosions. It is tentatively recommended that shallow source levels, where the surface reflection cannot be separated out, be determined by the method of the present study.

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SOURCE LEVELS OF SHALLOW UNDERWATER EXPLOSIONS

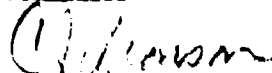
This report is part of a continuing study of the near-field acoustic characteristics of underwater explosions. Small explosive charges are frequently the sources used in underwater acoustic experiments, particularly those dealing with long-range propagation of low-frequency sound. Such acoustic experiments rarely include direct near-field measurement of the charge output for each shot; source level values are typically estimated quantities. Since there is no single, generally accepted method of estimating source levels at present, different users of explosion sources sometimes arrive at significantly different values of this important parameter.

This report presents idealized acoustic spectra for several explosive charge weight/depth configurations frequently used in underwater acoustics research studies. These spectra, which were derived from quasi-theoretical pressure-time signatures, have not yet been adequately validated. Additional work required to confirm or modify the idealized source level functions is outlined.

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ROBERT WILLIAMSON, II  
Captain, USN  
Commander



C. J. ARONSON  
By direction

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## SOURCE LEVELS OF SHALLOW UNDERWATER EXPLOSIONS

### 1. INTRODUCTION

1.1 Because of the widespread use of underwater explosions as acoustic sources for long-range propagation studies, the problem of the determination of explosion source energy levels for underwater explosions has become increasingly important in recent years. Transmission loss is perhaps the foremost problem in underwater acoustics research, and reliable source levels are needed if transmission loss is to be found from measurements of received signal level. For many explosion geometries, a more or less direct measurement of the source level is possible. When the explosive source is so shallow that the perturbations due to the water surface are not time-separable from the direct effects, no direct measurement of the source level can be made. An analytical approach, using quasi-theoretical, idealized pressure-time curves to determine the energy spectrum, can be useful for calculating the source levels of shallow explosions. One such calculation is the standard treatment by Weston.<sup>1\*</sup>

1.2 The work documented in this report uses idealized pressure-time curves which comprise a more detailed representation of the free water curves than those used by Weston. Using a digital computer, and fast Fourier transform techniques, the energy spectra are determined, and may be integrated in any desired analysis bands. This technique has been used to determine the source levels at a 100-yard reference range\*\* for several conditions of interest in order to demonstrate the method as well as to provide source levels for these conditions. These source levels are displayed graphically for frequencies up to 500 Hz and tabulated in octave and 1/3-octave bands for frequencies of 11 Hz to 11 kHz.

1.3 The discussion below includes a brief treatment of underwater explosion phenomenology related to the use of explosions as acoustic sources, as well as a discussion of the problems encountered in the use of explosive sources. Pertinent previous work on explosive source levels is reviewed and related to the present calculations. The details of the construction of the idealized pressure-time curves and the results of the analysis are given. Some preliminary comparisons of the present results with those of previously used techniques have been made and evaluated.

### 2. UNDERWATER EXPLOSION PHENOMENOLOGY

2.1 Many of the problems encountered in dealing with underwater explosions as acoustic sources are due to the complex nature of the explosion phenomena, and consequently, the pressure-time signal. The processes involved are fairly well understood. The following brief discussion will touch only on the points relevant to the pressure signal produced. The reader is referred to the book by Cole<sup>4</sup> and to Weston's paper<sup>1</sup> as good general references.

<sup>1\*</sup> Refers to references on page 24 and 25.

<sup>\*\*</sup> The 100-yard reference range is chosen following Weston (Ref. 1), Christian (Ref. 2), and others. The scaling of energy spectra from one range to another is discussed by Christian in Ref. 3.



2.2 At the time of detonation of an underwater explosive charge, the explosive is transformed by the detonation wave into a small sphere of gaseous explosion products at high pressure, and a shock wave is propagated outward. The gas globe, being at a pressure much higher than the surrounding medium, begins to expand in size. As the sphere expands, the pressure in the tail of the shock wave is reduced, corresponding to the decrease in pressure in the bubble. The bubble continues to expand until it reaches its maximum extent--at which time the transmitted pressure, having become negative with respect to ambient hydrostatic pressure, is at a minimum. The hydrostatic pressure is now greater than that in the bubble; consequently, the bubble begins to contract and is carried by momentum through the equilibrium volume to a very small minimum. A positive pressure pulse, the first bubble pulse, is emitted and the bubble expansion and contraction cycle begins again and may continue through as many as ten cycles. Each succeeding cycle takes a shorter time, and each succeeding bubble pulse has a lower peak pressure and less energy. Figure 1 shows a typical explosion pressure signal along with a depiction of the bubble size at various stages of the cycle.

2.3 The time interval between the shock wave and first bubble pulse, termed the first bubble period, is a sensitive function of charge weight and depth. If the effect of the boundaries of the water is neglected, the first bubble period,  $T_1$ , is given by the expression

$$T_1 = KW^{1/3}/Z^{5/6}, \quad (1)$$

where W is the charge weight, Z is the total hydrostatic head, and K is a coefficient depending on the explosive composition.\* Some representative values of  $T_1$  for given conditions are shown in Table 1 (see also Figure 2).

Table 1

First Bubble Period,  $T_1$ , for TNT (seconds)

| Charge Weight<br>(lb) | Charge Depth (ft) |      |       |
|-----------------------|-------------------|------|-------|
|                       | 50                | 500  | 5000  |
| 1                     | .11               | .024 | .0036 |
| 10                    | .24               | .050 | .0078 |
| 100                   | .51               | .11  | .017  |

2.4 The functional dependence of the various explosion pressure history parameters is discussed in section 5, below.

2.5 While the explosion bubble is pulsating, it may experience significant upward migration as a result of buoyancy. Thus, the explosion becomes a moving source, with succeeding bubble pulses originating at shallower depths. Bubble migration has been treated by Smay<sup>7</sup>. Calculations show that for the charge sizes and geometries considered in this report, migration is not a significant effect. For the 3-lb, 60-ft condition, the strongest migrating condition considered in this report, the bubble migrates about 2 feet to the 2nd bubble maximum. This is not considered a major influence on the source levels. For larger charges, at moderate depths, the effect is more important.

---

\* K may exhibit a slight dependence on charge depth for some explosives.

2.5 The explosion pressure signal is in general modified by the effect of the surface of the water. The reflected signal generally exhibits a phase change of  $180^\circ$  with respect to the direct arrival. For shallow explosions, this reflected signal is not time separable from the direct arrival. The relationship of the surface reflected pulse to the study of explosive source levels is discussed in the next section.

### 3. UNDERWATER EXPLOSIONS AS ACOUSTIC SOURCES

3.1 An underwater explosion is a broadband energy source, with considerable acoustic energy radiated over a broad range of frequencies. Explosives are generally inexpensive and produce a great amount of energy per dollar in any frequency band of interest. Underwater explosions may be easily detonated at any desired depth, without the need for connecting wires or cables. Explosive sources are non directional; they radiate a large amount of energy in all directions. This is both an advantage and a drawback of explosive sources. The explosion may radiate adequate energy in the desired direction for a particular application, but the fact that it radiates the same energy in all directions leads to high reverberation noise levels. Another difficulty is that the explosive source is not well suited to get repeated "looks" at the target in active sonar applications. In spite of these drawbacks, the explosive source has found wide usage in the acoustics community. The general subject of underwater explosions as acoustic sources is discussed by Weston<sup>1</sup> and Urlick.<sup>6</sup>

3.2 The source level of an underwater explosive charge gives a measure of the acoustic energy in a specified frequency band that is radiated by the charge to a particular point in space: it is a frequency-domain representation of the pressure-time function shown in Figure 1. In many cases, experimental conditions are such that the boundaries or the medium itself introduce perturbations in the pressure signal before the bubble pulses have subsided. Here we are concerned with such perturbations as those caused by the phase-shifted surface reflection. When the surface reflection is superimposed on the direct arrival, neither direct calculations of the source level from the measured pressure signal nor analogue measurement of the source level is possible. Because the surface reflection is not time-separable from the direct effects of the explosion, some manipulation, such as the frequency demodulation of Parkins,<sup>7</sup> is necessary before a true source level can be determined from recordings in which the rarefaction wave interferes with the direct wave. The time-domain treatment discussed here is another approach to the problem of accurately determining source levels of "shallow" explosions.

3.3 In the context of this report, "deep" explosions are those for which the surface reflection arrives after the bubble pulses have subsided. For these "deep" explosive sources, there is no confusion as to what is meant by source level. It involves the total acoustic energy radiated by the charge to some fixed range (such as 100 yards). For "shallow" sources, this concept remains valid; however, any measurements of the source levels of such "shallow" explosions unavoidably contain energy contributions due to the presence of the surface. Some workers have erroneously modified the above concept of source level to include surface reflection effects since no direct determination of "shallow" source levels can otherwise be made.

3.4 The time interval between arrivals of the direct and surface reflected wave is a geometric function independent of the charge weight. The bubble periods, and thus the duration of the direct arrival, depend upon the weight as well as the depth of the charge. It is the relationship of the time of arrival of the surface reflection to the duration of the direct pulse that determines whether source level measurements can be made: when the surface reflection occurs after the last detectable bubble period, direct source level measurements are possible.

3.5 Both 60-foot conditions of this report are those for which no direct source level measurements can be made. For the other, deeper depths (300, 500 and 800 feet), the observation point dictates the possibility of source level measurements. Under many experimental conditions, one may not have any latitude in selecting the observation point (such as when a fixed hydrophone system is being utilized).

3.6 An analytical method to determine "shallow" explosion source levels has been evolved and is the subject of sections 5-7 of this report. The method is also applied to several cases of "deep" explosions to provide source levels for cases in which no appropriate near-field measurements have been made.

#### 4. PREVIOUS WORK ON SOURCE LEVELS

4.1 The previous work on the close-in energy spectra of underwater explosions may be loosely classified into the two following categories:

1. Analytical treatments based on the Fourier analysis of simple idealized pressure waveforms (usually combinations of exponentials) and
2. Analysis of Experimental Data.

These two types of treatment will be discussed briefly below and examples provided.

#### 4.2 Analysis of Simple Waveforms.

4.2.1 These treatments take advantage of the fact that the shock wave from an underwater explosion has an initially exponential decay. The shock wave pressure-time history is treated as an exponential pulse,

$$P(t) = P_0 e^{-t/\theta} \quad (3)$$

where  $P_0$ , the peak shock wave pressure, and  $\theta$ , the decay constant, are usually evaluated from published empirical relationships. Treatments using only the shock wave in this manner are highly unrealistic as the energy contribution of the bubble pulses is very important. Therefore, bubble pulses are usually taken into account. Typically, the bubble pulse is idealized as a double exponential. The time of the peak bubble pressure is taken from experiment. Such simple idealized waveforms have the advantage of being amenable to solution for the energy spectrum in closed form. This allows insight into the variation of different spectral parameters with changes in the pressure-time parameters.

4.2.2 The major disadvantage of these treatments is that exponentials do not accurately model the explosion pressure pulse. While an actual explosion shock wave has an initially exponential decay, after a time of roughly 2 $\tau$ , the decay rate lessens markedly. The shock wave pressure becomes negative (relative to hydrostatic pressure) at a time strongly dependent on the depth of burst and contains a significant amount of energy in the negative phase. The bubble pulses, having negative pressure phases both before and after the bubble maxima, are also not well modeled by exponentials. Nonetheless, such treatments have proven both practical and convenient for determination of source levels.

4.2.3 The prime example of this type of work is the well-known treatment by Weston.<sup>1</sup> It should be noted that in his paper Weston presents two methods of determining source levels, an analytical method and one based on experimental data. He recommends the use of his experimental results as the more accurate method of obtaining source levels. It is apparent, however, from the nominal explosive conditions shown, that surface reflections were included in some of his data. For this reason, the following discussion will consider only the analytical formulation.\* Weston considers a pressure history consisting of a shock wave and two bubble pulses. He chose to separate his analysis into two parts, the low and the high frequencies. At low frequencies, the shock wave and bubble pulses are replaced by their respective impulses. In order to correct the residual impulse of his model, Weston introduces a negative impulse, corresponding to a steady negative pressure, which causes the total residual impulse to be zero. At higher frequencies, Weston shifts to an exponential shock wave followed by two double exponential bubble pulses. He uses a formula for summing the spectral contributions of the three pulses which neglects the interference patterns between the pulses. Weston's analysis includes wave form variations only through the bubble periods, and takes no account of the variation in energy and impulse in the first positive phase (shock wave) with source depth. Weston's analysis is discussed further in section 7.

4.2.4 Other examples of this type of analysis are given by Brumbach<sup>8</sup> and Slifko<sup>9</sup>. This latter work treats only the shock wave portion of the pressure time history.

### 4.3 Analysis of Experimental Data

4.3.1 The determination of source levels by the direct analysis of experimental pressure-time curves is a widely used procedure. The measured pressure histories are analyzed, either with analog or digital processing, to yield, more or less directly, the energy spectrum at a reference range. This method, while the most direct, has several drawbacks. The measurements are rarely made at the chosen reference range. Instead, various propagation models are used to extrapolate back to this range. Models varying from spherical spreading through refractive calculations and sophisticated finite amplitude analyses have been used. In addition, the desired source level is not generally identical with the energy spectrum of an experimental pressure history, even if recorded at the appropriate range.

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\* When other authors use Weston's source levels, it is often not made clear which of his formulations has been used.

The pressure history is subject to distortions, such as reflections from the bottom and surface of the water which are not part of the source level. In addition, localized effects such as bulk cavitation may have a profound effect on a measured spectrum, while being relatively unimportant at other ranges. Furthermore, the accurate recording of the pressure history causes serious experimental difficulties. High-quality broadband ( $\sim 0 - 20\text{kHz}$ ) recordings are difficult to achieve. Many experimenters record, therefore, only in limited frequency bands. This makes it very difficult to isolate the spurious effects mentioned above, with the consequence that some experimenters seem to have included such effects in their source levels.

4.3.2 A valuable effort in this direction is the paper by Christian<sup>2</sup>. This paper documents the measured spectra of a large number of charges. The recordings were high quality, broadband; the analysis was digitally performed. The results were filtered into octave bands and presented in a scaled form as a family of curves representing all charge weights at source depths from 300\* to 22,000 feet. By reducing the energy by the  $4/3$  power of the charge weight, Christian collapsed the data from various charge sizes at the same depth into a single curve. The recordings were carefully analyzed to eliminate spurious effects. Since the recordings of the data was done directly above the charge, refractive effects were eliminated. Theoretical considerations were used to separate the effects of charge depth and range to the measuring hydrophone. Christian's curves have been widely used to provide source levels in situations where no close-in measurements have been made, with several workers tending to substantiate her results, notably Kibblewhite and Denham<sup>11</sup>. On the other hand, Turner and Scrimger<sup>12</sup> report close-in measurements that do not agree with Christian's curves for some frequencies. This point is discussed further in section 7.4.

4.3.3 Other work along these lines has often concerned itself with the determination of the source spectrum for a condition of particular interest to the worker, such as Maples and Thorp<sup>13</sup>. Christian and Blaik<sup>14</sup> give a good general treatment based on experimental measurements.

4.3.4 While this type of work has proven generally useful, the direct determination of source levels for very shallow explosions is not possible, as pointed out in section 3. Efforts to demodulate the measured spectrum to remove the effect of the surface reflection in the frequency domain have been made<sup>7, 15</sup>. These efforts have generally been hampered by the lack of a realistic model for the surface reflected pressure. The reflected pressure history is not always, as is often assumed, simply the mirror image of the direct pressures with an appropriate time delay. As pointed out by Parkins<sup>7</sup>, scattering at the interface tends to destroy the coherency of surface reflected arrivals. The noise introduced by the scattering seriously erodes the simple ideal modulation when surface roughness is important. A more accurate model of the surface reflection process, taking surface scattering into account, would be needed for this method to prove more generally applicable. In some instances, however, the simple model may be acceptable (for example, see paragraph 7.6 and reference 15).

---

\* The 300-and 500-ft. curves of reference 2 are not based entirely on actual data at those depths. The curves are interpolated between the deeper curves and Stockhausen's shock wave data (reference 10) at high frequencies and extrapolated from the deeper curves at low frequencies.

4.3.5 Efforts to eliminate the surface reflection from measured pressure records by digitally processing them and setting all negative pressures equal to zero, as done in reference 13 is unrealistic in that the energy contribution of the negative pressure phases of the direct arrival, which may be appreciable at low frequencies, is eliminated.

4.4 Of the above methods, the use of idealized free water pressure-time curves appears to be the most promising for the determination of shallow explosive source levels. In the method of the present report, the best available information on explosive pressure-time histories has been utilized to construct quasi-theoretical pressure-time curves for several shallow explosion geometries of interest. We feel that these idealized pressure histories represent an improvement over those methods previously used. The details of their construction are given in the following section.

## 5. CONSTRUCTION OF IDEALIZED PRESSURE-TIME HISTORIES

5.1 Idealized pressure-time histories for several combinations of charge weight and depth have been constructed utilizing the best available information for deeper explosions and extrapolating to the shallower depths. The details of this procedure are given below.

The major body of information utilized was reported by Slifko<sup>16</sup>. This report documents the analysis of a large body of data gathered by the Naval Ordnance Laboratory (NOL). A total of 56 charges weighing 1, 8 and 50 lb were fired at depths from 500 to 14,000 feet. Pressure-time records were obtained with an LC32 hydrophone located directly above the charge and 185 feet from the surface. Semi-empirical equations were derived which give the various parameters of the pressure-time record as functions of depth, range and charge weight. Theoretical considerations were utilized to separate the effects of range and charge depth.

5.2 In addition, information from other available sources was incorporated into the theoretical curves used for this study. The various parameters used as inputs to the construction of the idealized pressure histories are indicated in Figure 2. Table 2 gives the functional forms and sources for these parameters.

Table 2

## FUNCTIONAL DEPENDENCE OF PRESSURE-TIME PARAMETERS

| Parameter | Functional Dependence                            | Source                  | Ref                       |    |
|-----------|--|-------------------------|---------------------------|----|
| $P_0$     | $2.08 \times 10^4 (W^{1/3}/R)^{1.13}$            | Slifko                  | 16                        |    |
| $P_{min}$ | $-77 Z^{1/3} W^{1/3}/R$                          | Slifko                  | 16                        |    |
| $P_1$     | $3300 W^{1/3}/R$                                 | Slifko                  | 16                        |    |
| $P_2$     | $.22 P_1$  | Unpublished data at NOL |                           |    |
| $P_3$     | $.10 P_1$  | Unpublished data at NOL |                           |    |
| $P_4$     | $.03 P_1$  | Unpublished data at NOL |                           |    |
| $\theta$  | $5.8 \times 10^{-5} W^{1/3} (W^{1/3}/R)^{-0.22}$ | Arons                   | 18                        |    |
| $T_{pp}$  | $= \frac{KW^{1/3}}{5/6 Z}, K =$                  | .555                    | Slifko                    | 16 |
| $T_{np}$  |  | 3.10                    | Slifko                    | 16 |
| $T_{bp}$  |  | 1.10                    | Slifko                    | 16 |
| $T_1$     |  | 4.34                    | Slifko                    | 16 |
| $T_2$     |  | 3.06                    | Arons, Slifko, and Carter | 17 |
| $T_3$     |  | 2.48                    | Arons, Slifko, and Carter | 17 |
| $T_4$     |  | 2.31                    | Arons, Slifko, and Carter | 17 |

All pressures in psi and times are in seconds

R = slant range, ft.

W = charge weight, lbs.

Z = charge depth + 33, ft.

5.3 The shock wave was constructed initially exponential with a decay rate,  $\alpha$ , as given by the similitude equation of Arons<sup>18</sup>. The exponential decay was continued out to approximately 2A. The impulse in the first positive phase (shock wave) and in the first bubble pulse positive phase was calculated from the relations given by Slifko<sup>16</sup>. We had no information on the impulse in the later bubble pulses and the minimum pressure of the later negative phases. From inspection of unpublished data, it was found that the maximum underpressure in the successive negative phases tended to become smaller. These parameters did not completely define the pressure signature, but it was felt that the uncertainties in the later states would not be of primary importance. The pressure-time curves were hand drawn according to the information given in Table 2 as described above. The records were digitized by use of the Telereadex and were analyzed digitally using NOL's CDC 6400 computer.

5.4 The first stage of the digital analysis was checking the digitized waveform for compliance with the information inputs. Minor modifications were made to the first and second positive phases to bring the impulse in these phases to within 10% of those calculated from Slifko's relationships. The negative phases were modified so as to produce near zero residual impulse out to the fourth bubble pulse.

5.5 The analysis was performed using a system of computer codes developed at NOL and known by the acronym, MR. WISARD. The package is described in detail in Reference 19. Using MR. WISARD, the spectra were computed using the fast Fourier transform technique of Cooley and Tukey<sup>20</sup>, with a bandwidth of about 1 Hz. The spectra were also integrated in octave and 1/3-octave bands (the bands used are indicated in Table 3) to facilitate comparison with spectra determined by analogue methods. The filter bands used were ideal (not physically realizable) in that they passed all the energy between the limiting frequencies and no energy from outside those frequencies.

5.6 In order to make comparisons with measured data for the shallow conditions an idealized surface reflection was introduced for some parts of the analysis. The reflection was assumed to cause a 180° phase shift with no losses at the interface. The amplitude of the reflected pulse was adjusted for the difference in travel distance between it and the direct arrival. As suggested by Christian<sup>17</sup>, the surface reflection was put in as a modification in the time domain, rather than a modulation of the energy spectrum. In our particular computational scheme, the effort required to modify the time domain functions was far less than that needed to correct for surface reflections in the frequency domain.

5.7 These idealized pressure-time records represent the best available synthesis of empirical and theoretical knowledge available to the authors at this time. The extrapolation from deeper to relatively shallow depths represents a major uncertainty. In the absence of a good theoretical treatment of shallow explosions, it was felt that the above represented the most promising approach to the problem. The results and comparisons of the following two sections shed light on the validity of the entire procedure.



## 6. RESULTS

6.1 Idealized pressure-time curves have been constructed using the methods detailed in the previous section for several conditions of interest. The MK 61 Signal Underwater Sound (SUS) charge contains 1.8 lb of explosive material (TNT) and is detonable at 60 ft. and 800 ft. The SUS MK 57-0 and MK 82-0 also contain 1.8 lb of TNT and have nominal firing depths of 800 ft. and 60 or 300 ft., respectively. To obtain source energy levels for these devices, pressure-time records for 1.8 lb of TNT detonated at depths of 60, 300 and 800 ft. were constructed. For the 60-foot condition, no direct measurement of the source level is possible. For the 300-foot, 1.8-lb source, with a 100-yard reference range, the angular position of the receiver may vary the arrival time of the surface reflection from being coincident with the direct arrival directly above the charge at the reference range to being between the third and fourth bubble pulses directly below the charge. Hence, direct measurement of the source levels will generally appear to exhibit directivity. The deeper 800-foot, 1.8-lb condition offers no problem since the direct and surface reflected arrivals are easily time separable. This condition is included for completeness and for comparison purposes.

6.2 In addition to the 1.8-lb charge weight, a weight of 3 lb was also treated at depths of detonation of 60 and 500 feet. These conditions were included because they have been used in important oceanic acoustic experiments. The 500-foot, 3-lb source which offers no apparent source directivity is included for completeness, while the 60-foot, 3-lb source must be treated by the method developed here for shallow charges.

6.3 To summarize, the five conditions calculated were 1.8 lb at 60, 300, and 800-foot depths and 3 lbs at 60 and 500-foot depths. Two representative pressure-time curves for 3 lb at 60 feet and 1.8 lb at 800 feet, are shown in Figures 3 and 4.

6.4 Energy spectra were computed for the five conditions and are shown in Figures 5-9. These spectra were computed with a resolution bandwidth of approximately 1 Hz. Each spectrum shows a complex pattern of peaks and nulls, the primary features of which may be related easily to the time domain characteristics. The highest peak occurs at the bubble fundamental frequency\*, and succeeding peaks at approximately higher harmonics of this frequency. The dominance of the first bubble pulse in the shape of the spectrum is due to the fact that the first bubble pulse contains significantly more energy than the following pulses. The later pulses do have an effect on the spectrum, as they introduce secondary interference patterns (which are conspicuous at the lower harmonics) and shift the peak locations at higher frequencies.

6.5 Although the source level at a range of 100 yards is fully defined by the detailed narrow-band spectra of Figures 5-9, many workers in ocean acoustics use frequency bands of 1/3-octave and 1 octave in their analyses. We have, therefore, integrated the energy spectra in these energy bands, and present the results below.

6.6 Table 3 gives the energy distribution for 1/3-octave bands and Table 4 gives it for the octave bands for the five conditions that have been studied. These

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\* The reciprocal of the time of the first bubble pulse.

values are graphically displayed in Figures 10-14. The energy rises from low values at low frequency, to a maximum near the bubble pulse fundamental frequency. It falls off thereafter, and at higher frequencies approaches the 6 db/octave falloff predicted for exponential waves. This is due to the dominance of the shock wave at high frequencies. The relative smoothness of the octave band energy is due to the fact that the integration bands are generally wide enough to average out the peaks and nulls present in the detailed spectrum (1 Hz bandwidths) of Figures 5-9. The 1/3 octave analysis shows some indication of the detailed structure of the spectrum. It should be pointed out that the apparent irregularity of the 1/3-octave spectra does not represent spurious behavior. The low frequency analysis bands are generally narrow with respect to the features of the detailed spectra, and may contain a peak, a null or any combination of peaks and nulls. Therefore, the energy may change in a seemingly sporadic way from band to band. At higher frequencies, as the bands become wider, they encompass a number of peaks and nulls of the spectrum. When the energy is integrated in these bands, the abrupt changes are smoothed out, and the spectrum approaches the smooth behavior of the octave spectra. Hence, the apparent irregularity of the spectra in 1/3-octave bands is seen to be caused by the relationship of the analysis bands used to the features of the detailed spectrum. The fluctuations are real, and represent genuine fluctuation in the source level with frequency.

6.7 In the figures showing the 1/3- and octave band spectra, the data points have been connected with straight lines. This has been done for convenience in demonstrating the variation in energy from band to band. It is important to note that these figures may not generally be used to interpolate between bands. For example, suppose we wish to determine the source level for a 60-ft 3-lb TNT charge at 25 Hz. The detailed spectrum of Figure 5 shows a null at this frequency, with energy of about 4 db. The 1/3-octave energy calculation at 25 Hz in Figure 10 is 19.5 db. Both these numbers are correct, the former giving the energy at 25 Hz for an approximate 1 Hz bandwidth, and the latter, the average energy in a 1/3-octave band with a geometric mean frequency of 25 Hz. These numbers would be expected to be different, due to the difference in the physical quantities represented. Again, though they differ greatly, both numbers are valid. In regions of relatively slow variation in the octave spectrum, say for frequencies at least twice the bubble fundamental, linear interpolation may give an indication of the energy to be expected in an octave band centered at the given frequency. This would only be a rough approximation. Linear interpolation at lower frequencies, however, would be extremely dubious.

6.8 If one were using the octave spectrum of Figure 10 in an attempt to determine the energy at 25 Hz--regardless of whether he is interested in a band of 1-Hz, 1-octave or 1/3-octave, he would not be able to extract from this figure the information desired. He might erroneously interpolate linearly between the bands centered at 16 Hz and 31.5 Hz to obtain 21 db--a value having no real meaning. From the exercise in this and the previous paragraph, three different values for energy at 25 Hz have been obtained. The first two, 4 db and 19.5 db, refer to the energy in a 1-Hz band centered at 25 Hz and the energy in a 1/3-octave band centered at 25 Hz, respectively. The 21 db at 25 Hz of this paragraph has no correct interpretation. It is clear that the source level for a particular application must be carefully determined and interpreted in terms of the appropriate bandwidth analysis used.

TABLE 3  
SOURCE LEVELS IN 1/3-OCTAVE BANDS-ENERGY IN db re 1 erg/cm<sup>2</sup>/1/2  
Range 100 yds

| Frequency Band (Hz) | 1.8 lb at 800' | 1.8 lb at 300' | 1.8 lb at 60' | 3 lb at 60' | 3 lb at 500' |
|---------------------|----------------|----------------|---------------|-------------|--------------|
| 11-14               | -2.1           | 6.9            | 23.0          | 23.8        | 0.5          |
| 14-18               | 0.5            | 13.9           | 19.0          | 22.4        | 8.5          |
| 18-22               | 4.7            | 17.4           | 18.7          | 22.3        | 15.5         |
| 22-28               | 9.5            | 20.7           | 20.0          | 19.2        | 20.6         |
| 28-35               | 13.1           | 20.4           | 17.2          | 20.3        | 23.3         |
| 35-45               | 15.8           | 17.4           | 16.6          | 19.6        | 21.1         |
| 45-56               | 18.6           | 15.7           | 14.9          | 17.5        | 14.9         |
| 56-71               | 17.9           | 14.0           | 16.0          | 18.2        | 16.3         |
| 71-90               | 13.3           | 13.5           | 14.0          | 16.3        | 14.7         |
| 90-112              | 14.2           | 13.3           | 13.7          | 16.0        | 15.1         |
| 112-140             | 11.1           | 11.5           | 11.7          | 15.2        | 14.3         |
| 140-180             | 11.8           | 11.5           | 10.3          | 13.9        | 13.7         |
| 180-224             | 9.5            | 10.1           | 9.5           | 13.0        | 13.4         |
| 224-281             | 8.5            | 9.1            | 8.6           | 12.0        | 11.5         |
| 281-355             | 7.1            | 8.3            | 7.6           | 10.5        | 10.7         |
| 355-447             | 7.6            | 7.2            | 6.6           | 9.3         | 10.0         |
| 447-561             | 6.4            | 6.2            | 5.5           | 8.2         | 9.4          |
| 561-710             | 5.3            | 4.9            | 4.2           | 7.1         | 8.3          |
| 710-894             | 4.1            | 3.7            | 2.6           | 5.9         | 7.0          |
| 894-1118            | 3.0            | 2.5            | 1.4           | 4.8         | 5.3          |
| 1118-1414           | 1.7            | 1.3            | .7            | 3.3         | 3.4          |
| 1414-1789           | -0.1           | -0.3           | -5.5          | 1.4         | 1.6          |
| 1789-2236           | -2.1           | -2.0           | -2.7          | -5          | 0.1          |
| 2236-2806           | -4.0           | -3.8           | -4.9          | -2.2        | -1.8         |
| 2806-3550           | -5.6           | -5.4           | -6.7          | -3.9        | -3.6         |
| 3550-4472           | -7.2           | -7.1           | -8.0          | -5.7        | -5.6         |
| 4472-5612           | -9.6           | -9.0           | -9.2          | -7.6        | -7.8         |
| 5612-7100           | -12.0          | -11.6          | -10.4         | -9.5        | -9.9         |
| 7100-8950           | -14.6          | -14.0          | -12.2         | -11.3       | -11.9        |
| 8950-11225          | -16.2          | -15.7          | -14.0         | -12.9       | -13.9        |

TABLE 1.

SOURCE LEVELS IN OCTAVE BANDS-ENERGY IN db re 1 erg/cm<sup>2</sup>/Hz

| Frequency Band (Hz) | Range 100 yds  |                |               |              |
|---------------------|----------------|----------------|---------------|--------------|
|                     | 1.8 lb at 800' | 1.8 lb at 300' | 1.8 lb at 60' | 3 lb at 500' |
| 11-23               | 2.9            | 15.4           | 20.2          | 22.9         |
| 23-45               | 14.0           | 19.4           | 17.9          | 19.7         |
| 45-89               | 16.8           | 14.4           | 15.0          | 17.3         |
| 89-177              | 12.4           | 12.1           | 12.0          | 15.0         |
| 177-354             | 8.3            | 9.0            | 8.5           | 11.7         |
| 354-707             | 6.3            | 6.1            | 5.3           | 8.2          |
| 707-1414            | 2.9            | 2.4            | 1.5           | 4.6          |
| 1414-2828           | -2.1           | -2.1           | -2.7          | -0.5         |
| 2828-5657           | -7.5           | -7.3           | -8.1          | -5.8         |
| 5657-11314          | -14.3          | -13.8          | -12.3         | -11.3        |

## 6.9 Effect of Small Variations in Burst Depth

6.9.1 It is a fact of experimental life that explosions often do not detonate at precisely their nominal depths. The depth span to be expected for SUS charges are 50-70 feet for a nominal 60-ft burst<sup>21</sup>, and 700-900 feet for a nominal 800-ft burst<sup>22,23</sup>. Approximately the same percentage variation in burst depth would probably occur with other nominal depth settings. These seemingly small variations in burst depth may have a significant effect on the source level in certain of the 1/3-octave and octave bands. In the time domain, the primary effect of depth variations is a change in the bubble periods and other characteristic times, which all vary as  $Z^{-5/6}$ , where  $Z$  is the charge depth + 33 ft, (see Table 2). The variation of the minimum pressure in the first negative phase is considered a second order effect.

6.9.2 In the frequency domain, the location of the peak at the bubble fundamental is shifted, and the energy at the peak decreases with increasing burst depth. Figure 15 shows the detailed spectra for 1.8 lbs of TNT at burst depths of 700, 800, and 900 feet. The bubble fundamental frequencies for these depths are 46, 51, and 56 Hz, respectively. The pattern of variation around the first peak in the spectrum is generally repeated at the succeeding peaks, but the variation is increasingly masked by the secondary interference patterns. The consistent variation of energy with burst depth near the bubble fundamental may be discerned in the 1/3-octave band analysis. The differences in energy,  $\Delta E$ , between the source level at 800 feet and that 700, 750, 850, and 900 feet are tabulated in Table 5. Up to about 50 Hz, the 1/3-octave band levels display a fairly consistent variation with depth, the energy level decreasing for increasing burst depth. Above this frequency, the pattern of variation seems more random. Table 6 gives a similar tabulation for a nominal 60-foot burst of a 1.8-lb TNT charge. The bubble fundamental frequency (~8 Hz for a 60-ft depth) is too low to be of practical importance in most applications. The variations depicted, then, are all for frequencies considerably above the fundamental, and have the same apparently random character as the higher frequency variations of Table 5.

6.9.3 The variation in source level caused by the above variations in depth may be as high as 6.1 db (in the 20 Hz 1/3-octave band) for the 800 foot nominal depth, and as much as 2.5 db (in the 31.5 Hz 1/3 octave band) for the 60-foot nominal depth. These variations may be quite significant in some applications. The implication here is that the depth of burst must be accurately determined in order for the source level to be given with reasonable accuracy. Merely using the nominal depth may lead to considerable, seemingly disproportionate, errors (6 db represents a factor of 4 in energy).

## 6.10 Surface Reflections

6.10.1 As mentioned above, apparently some workers have erroneously included surface reflections as part of the source level. The effect of surface reflections on the energy spectrum has been described in detail by Christian<sup>24</sup>. Briefly, the effect is to impose a modulation, due to the reflection, on the spectrum of the direct arrival.

6.10.2 As an example of this, take the case of a 3-lb TNT charge detonated at 50 feet. The maximum time delay between the direct and surface reflected arrivals is 24 msec. This maximum delay is obtained when the measurement is made directly beneath the charge. Since this maximum reflection time is shorter than the length

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TABLE 5

DEPTH VARIATION FOR AN 800-FOOT, 1.8-LB SOURCE

$\Delta E (E_1 - E_{800})$  (db re 1 erg/cm<sup>2</sup>/Hz)  
Range 100 yds

| 1/3 Octave Band<br>Center Frequency | Source Depth |      |      |      |
|-------------------------------------|--------------|------|------|------|
|                                     | 700'         | 750' | 850' | 900' |
| 12.5                                | 1.8          | .9   | -.7  | -1.4 |
| 16                                  | 2.8          | 1.3  | -1.1 | -2.0 |
| 20                                  | 3.3          | 1.5  | -1.5 | -2.8 |
| 25                                  | 2.8          | 1.4  | -1.4 | -2.7 |
| 31.5                                | 2.2          | 1.1  | -1.0 | -2.0 |
| 40                                  | 2.4          | 1.2  | -1.0 | -1.8 |
| 50                                  | 1.2          | .7   | -.9  | -1.7 |
| 63                                  | 1.2          | -.4  | -.1  | 0.0  |
| 80                                  | 2.0          | -1.0 | .1   | 0.1  |
| 100                                 | 0.0          | .1   | -.2  | -.5  |
| 125                                 | 1.7          | -1.3 | .8   | 1.1  |
| 160                                 | 2.2          | -.4  | -.2  | -1.2 |
| 200                                 | 1.0          | -.5  | -1.6 | -2.3 |
| 250                                 | 1.2          | .1   | 0.0  | .3   |
| 315                                 | 1.6          | 1.7  | .8   | 1.1  |
| 400                                 | .5           | .4   | -.4  | -.3  |
| 500                                 | .4           | .3   | -.1  | -.5  |
| 630                                 | .6           | .4   | -.1  | -.2  |
| 800                                 | .8           | .5   | .2   | -.1  |
| 1000                                | .7           | .5   | .2   | 0.0  |

TABLE 6

DEPTH VARIATION FOR A 60-FOOT, 1.8-LB SOURCE

$\Delta E (E_d - D_{60})$  (db re 1 erg/cm<sup>2</sup>/Hz)

Range 100 yds

| 1/3-Octave Band<br>Center Frequency | Source Depth |     |      |      |
|-------------------------------------|--------------|-----|------|------|
|                                     | 50'          | 55' | 65'  | 70'  |
| 12.5                                | -.8          | .4  | .4   | .7   |
| 16                                  | -.9          | .5  | .0   | .4   |
| 20                                  | -.4          | -.7 | .2   | -.1  |
| 25                                  | -.5          | .0  | -.6  | -1.8 |
| 31.5                                | 1.4          | 1.2 | -1.2 | -1.1 |
| 40                                  | .8           | .1  | -.4  | .0   |
| 50                                  | 2.2          | 1.1 | 1.5  | 1.5  |
| 63                                  | -.5          | -.5 | -1.0 | -1.7 |
| 80                                  | .6           | 1.1 | .6   | .3   |
| 100                                 | -.2          | -.3 | -.9  | -.2  |
| 125                                 | .7           | .3  | .2   | .2   |
| 160                                 | .7           | .4  | -.1  | .0   |
| 200                                 | .4           | .3  | -.2  | -.3  |
| 250                                 | .4           | .2  | -.1  | -.3  |
| 315                                 | .3           | .1  | -.3  | -.4  |
| 400                                 | .5           | .2  | -.1  | -.3  |
| 500                                 | .4           | .3  | -.1  | -.2  |
| 630                                 | .3           | .2  | .0   | -.1  |
| 800                                 | .3           | .2  | .0   | .1   |
| 1000                                | .6           | .3  | -.1  | -.3  |

of the pulse train, the reflection is not separable from the direct arrival. Figure 16 shows the detailed spectra for the ideal pulse with no reflection and with a reflection at 24 msec. As may be seen, the reflection imposes a modulation on the free water spectrum, with nulls at approximately integral multiples of the inverse of the surface reflection time ( $\frac{1}{24 \text{ msec}} = 41.7 \text{ Hz}$ ). The effect of this

modulation on the 1/3-octave band spectra is shown in Figure 17. The errors introduced by including the surface reflection as part of the source level are large and seemingly random. For example, the surface reflection adds about 4 db at 20 Hz, and subtracts about 5 db at 40 Hz. No simple correction of a constant number of db will suffice. This example indicates the primary importance of eliminating the surface reflection when a source level is to be obtained from a measured pressure-time history.

6.11 In the following section, some preliminary comparisons of the results of the present study with previous calculations and measured data are presented.

## 7. COMPARISONS WITH PREVIOUS WORK

7.1 In order to establish the validity of the shallow (60 foot) source levels of the present study, the theoretical model should be compared with appropriate measurements. Of course, a surface reflection would have to be added to the idealized pulse in order to make such comparisons. For such a comparison to be realistic, however, experimental conditions (e.g., accurate charge and receiver locations, equipment response characteristics) must be known in greater detail than is reported for available published data. Efforts to obtain existing unpublished data for analysis and comparison have been unsuccessful up to the time of this writing.

7.2 In the absence of such shallow data, comparisons with deeper data have been made. These comparisons verify the accuracy of the method used in this study, except (as indicated above) for the vital extrapolation of semi-empirical pressure time relations from deep (>500 feet) to shallow depths, as discussed in section 5. The extrapolation has been partially evaluated by comparisons with data at a 300 foot charge depth, but further comparisons will be necessary.

7.3 The present results are compared with those of Christian<sup>2</sup> in Figures 18 through 20. The 300 and 500 foot curves are from reference 2, while the 800 foot source level curve is previously unpublished data<sup>25</sup>. The idealized source spectra have been integrated in the same octave bands and scaled in the same manner as done by Christian. The maximum difference shown on these three figures is ~ 2.5 db with a mean absolute difference of ~ 1 db. Considering that Christian's 300- and 500-foot curves are extrapolations that apparently fail to allow sufficient energy contributions from the bubble phases\*, the results derived from this study are believed to be more correct for these shallower conditions than the published ones. Agreement between the present study and Christian's 800-foot data is excellent (Figure 20).

7.4 The data of Turner and Scrimger<sup>12</sup> offer further valuable comparisons. Their source level data for 1 lb of pentolite at depths of 295 and 555 feet have been compared with the idealized results of this study, and this comparison is

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\* See footnote on page 6.



presented in Table 7. The energy spectra of the idealized pressure records were computed after adjusting the pressure and time scales to those of a 1-lb TNT charge at the same ranges and depths. No allowance for the different explosive compositions was included in the calculation. For both test conditions, Turner and Scrimger show more energy at low frequencies and less at high frequencies than the present results. This is the same general pattern as shown in their published comparison with Christian's source level curves, though the magnitude of the differences are reduced in the present study. At 295 feet, the maximum deviation in Table 7 is 2.6 db. (The mean absolute deviation is 1.6 db.) This is considerably better than the 555-foot comparison, which has a maximum deviation of nearly 5 db. (The mean absolute deviation is 2.6 db.) Part of these differences may be attributable to the fact that pentolite has about 20% more shock wave energy than TNT but the major differences, especially at 555 feet, are unexplained. Differences in processing techniques, such as recording, range correction, digitization and computer analysis techniques are possible sources of these large deviations.

7.5 Several additional samples of data available at NOE have been analyzed for comparison with the idealized spectra. These data are part of another collection reported by Christian and others in a classified report and will be referred to as Squaw shots. Three samples of data from charges at a nominal depth of 800 feet are compared with the present results in Figure 21. The actual depths were 835, 907, and 876 feet. The recordings were made at ranges of 1200, 1000, and 260 yards respectively from the three shots. The sources were Mk 61 SUS charges (1.3 lb of TNT). The three records were scaled to an 800-foot source depth by adjusting the time scale; spherical spreading was used to correct to a 100-yard reference range. The energy was computed in 1/3-octave bands. The source levels for these three Squaw shots and for the idealized 800-foot curve (in 1/3-octave bands) are shown in Figure 21. Note that the scatter among the three Squaw shots is on the order of 1 to 2 db. The differences between the Squaw shots and the idealized curve are generally less than 2 db up to about 500 Hz. The differences above 500 Hz have not been resolved at this time, but our assumption of simple spherical spreading to correct for range differences probably accounts for them. The general pattern of irregularity shown in the data is well modeled by the idealized curve. The dashed curve also shown in Figure 21 is discussed in section 7.9 below.

7.6 A fourth Squaw shot was selected to provide a more shallow comparison. This was another Mk 61 SUS detonated at 300 feet. The broad-band pressure-time record shows a surface reflected arrival at 24.7 msec after the direct arrival. For comparison, a 180° phase shifted surface reflection was introduced into the idealized pressure-time curve at 24.7 msec. Both theoretical and measured pressure histories were analyzed in 1/3-octave bands and are shown in Figure 22. The agreement again is good; the general pattern of peaks and nulls is especially close. Considering the crude nature of the surface reflection model used, the agreement is excellent. This is an indication that the extrapolation of our idealized waveform parameters from > 500 to 300-ft source depth is a valid procedure. The further extrapolation, to a 60-foot source depth, remains uncertain.

7.7 Since the calculation of Weston has long been the standard one for source levels, a comparison between the present results and those of Weston is included here. Before presenting this comparison, some comments regarding the differences between the two studies are in order.

TABLE 7

COMPARISON OF TURNER AND SCRIDGER'S RESULTS (Ref. 12) WITH THE  
PRESENT STUDY

(Energy in db re 1 erg/cm<sup>2</sup>/Hz)  
Range 100 yds

| <u>Freq. Band</u>       | <u>1 lb at 295'</u> |                      |          | <u>1 lb at 555'</u> |                      |          |
|-------------------------|---------------------|----------------------|----------|---------------------|----------------------|----------|
|                         | <u>Ref 12</u>       | <u>Present Study</u> | <u>Δ</u> | <u>Ref 12</u>       | <u>Present Study</u> | <u>Δ</u> |
| 20-40                   | 17.6                | 16.4                 | 1.2      | 13.0                | 8.7                  | 4.3      |
| 40-80                   | 14.7                | 12.5                 | 2.2      | 17.7                | 14.0                 | 3.7      |
| 80-160                  | 12.4                | 10.1                 | 2.3      | 13.6                | 9.0                  | 4.6      |
| 160-370                 | 9.4                 | 6.8                  | 2.6      | 9.9                 | 6.2                  | 3.7      |
| 370-640                 | 6.0                 | 4.1                  | 1.9      | 6.4                 | 4.3                  | 2.1      |
| 640-1280                | -.9                 | .3                   | -1.2     | -.5                 | 1.2                  | -1.7     |
| 1280-2560               | -4.8                | -3.6                 | -1.2     | -4.8                | -3.4                 | -1.4     |
| 2560-5120               | -10.1               | -8.7                 | -1.4     | -10.0               | -8.6                 | -1.4     |
| 5120-10240              | -15.0               | -14.4                | -.6      | -14.9               | -14.4                | -.5      |
| Mean absolute deviation |                     |                      | 1.6      |                     |                      | 2.6      |

7.8 As mentioned above, in the low frequency region, Weston uses an impulse formula to calculate the energy spectrum. Both the shock wave positive impulse and that of the first two bubble pulses are included, with a negative impulse equal to their sum added to achieve zero residual total impulse. The formulas used for the impulse were the best available at that time, but must be examined carefully in the light of subsequent work. Weston uses the shock wave positive impulse formula reported by Arons<sup>18</sup>. This formula was synthesized from experimental records of large charges (25-76 lb) fired in shallow water. Since the time of arrival of the surface reflection was very short in relation to the duration of the shock wave positive phase, the impulse could not be determined for the entire shock wave phase of the pulse. Although the point is not made clear by Arons, comparison of the formula with that reported by Arons in reference 26 indicates that the impulse represented by Arons' formula is integrated out to only 6.7a, as was customary at that time. Arons and Yennie<sup>27</sup> indicate that the impulse at 6.7a is something less than 60% of the total impulse of the shock wave for a 250-foot deep shot. In addition, the results of Arons show no depth variation in the impulse, since all of Arons' data were from approximately the same depth. The shock wave impulse formula of Slifko, as used in this study, represents the total impulse of the first positive phase (out to the first zero pressure crossing), and indicates a significant depth variation. The ratio of Slifko's to Arons' impulse formula as a function of charge depth is shown in Figure 23.\* Shallower than 500 feet, Slifko's formula represents an extrapolation from his measured data. This extrapolation, while uncertain, is probably a more reasonable indication of the impulse at shallow depths than Arons' formula on two counts:

1. Arons' formula represents the impulse out to only 6.7a while Slifko's is for the entire shock wave positive phase, and
2. Slifko takes depth variation into account while Arons does not.

Aron's and Slifko's formulas agree at 500-ft depth. At more shallow depths, Slifko gives more impulse; at deeper depths, less impulse. The impulse formula used by Weston for the first bubble pulse impulse is also quite different from that used in the present study\*\*. One would expect considerable differences between Weston's analysis and the present study in the low frequency region. It is felt that the waveforms of the present study allow a more complete and accurate representation of spectral energy content than the approximations of Weston.

7.9 In order to make comparisons, Weston's analytical calculation has been repeated for the 1.8-lb SUS charge at depths of 60 and 800 feet. The results have been integrated in 1/3-octave bands and are compared with the corresponding present results. The 1/3-octave band differences are indicated in Table 8; the detailed spectra are compared in Figures 24 and 25. For 1.8-lb at 60 ft condition, the present results indicate considerably more energy--as much as 8 db in the very low frequency bands. This is largely due to the different impulse formulas used. In high frequency bands, Weston shows 2 to 3 db more energy for the 60-ft source. For an 800-ft source Weston's formulation gives much more energy at very low frequencies--again due largely to the different impulse formulas; at higher frequencies, Weston shows 1 to 3 db more energy. The differences for both burst depths are significant. To resolve this discrepancy, comparisons with data are needed. As noted above, we have no suitable 60-ft data for comparisons. For the

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\* The factor  $\left(\frac{R}{W^{1/3}}\right)^{.03}$  has been neglected in this ratio.

\*\* Weston's equation 7 contains a typographical error. The exponent of W should be 2/3.

TABLE 8

1/3-OCTAVE BAND COMPARISON OF CALCULATIONS USING WESTON'S  
ANALYTICAL FORMULATION WITH THOSE OF THE PRESENT STUDY  
Range 100 yds  
 $\Delta E$  ( $E_{\text{present results}} - E_{\text{Weston}}$  in db re 1 erg/cm<sup>2</sup>/Hz)

| <u>1/3-Octave band<br/>center frequency</u> | <u>1.8 lb at 60'</u> | <u>1.8 lb at 800'</u> |
|---|----------------------|-----------------------|
| 12.5  | 8.0                  | -5.4                  |
| 16  | 2.3                  | -4.9                  |
| 20  | 4.2                  | -2.1                  |
| 25  | 2.1                  | 2.4                   |
| 31.5  | -.1                  | 5.2                   |
| 40  | .8                   | 2.3                   |
| 50  | -1.0                 | 1.0                   |
| 63  | -1.1                 | 2.2                   |
| 80  | -2.2                 | 0.9                   |
| 100   | -2.5                 | -0.7                  |
| 125   | -2.4                 | -1.1                  |
| 160   | -2.4                 | -1.1                  |
| 200   | -2.6                 | -2.4                  |
| 250   | -2.4                 | -2.6                  |
| 315   | -2.3                 | -3.0                  |
| 400   | -2.1                 | -1.7                  |
| 500   | -2.2                 | -1.5                  |
| 630   | -2.2                 | -1.3                  |
| 800   | -2.4                 | -1.1                  |
| 1000  | -2.2                 | -0.7                  |

800-ft source, Weston's values have been plotted in Figure 21, along with the three Squaw shots and the idealized 800-foot source level curves of the present study. This figure indicates that whenever the two quasi-theoretical source level curves differ significantly, Weston's analytical curve shows less satisfactory agreement with the measured data. This, together with the generally satisfactory comparisons with available data, is some indication that the source levels of this study represent an improvement over Weston's analytical formulation.

## 8. SUMMARY AND CONCLUSIONS

8.1 The computational methods developed in this study can be used to estimate acoustic source levels for small charges (weighing up to a few lb) detonated at depths down to about 1000 ft. Results for very shallow sources (e.g. 60 ft) will remain less reliable than those for deeper sources until we acquire suitable data to compare with the idealized shallow-burst model. The quasi-theoretical pressure-time histories and their corresponding energy levels reported here are believed to be the best estimates available at this time.

8.2 Detailed plots (analysis bandwidth approximately 1 Hz) of idealized source spectra at 100 yards range are shown in Figures 5-9 for the following configurations: 3-lb charges at 60 and 500-ft detonation depths, and 1.8-lb charges at 60, 300, and 800-ft detonation depths. Estimated source levels in 1/3-octave and 1-octave bandwidths are summarized in Tables 3 and 4, respectively. The 1.8-lb results apply to underwater sound signals (SUS charges) Mk 57, Mk 61, and Mk 82 with appropriate depth settings; the 3-lb charge results correspond to test conditions used in an extensive research study.

8.3 In some instances considerable difference was found between the theoretical source levels presented here and previous measurements. For example, although overall agreement with the data of Christian (Refs. 2 and 25) was good, deviations of up to 3 db occurred in certain frequency bands (paragraph 7.3, 7.5, 7.6). Significant differences from Christian's data can be resolved, since all necessary information about methods used in data recording and processing is available at this Laboratory. Unfortunately, the same is not true for the data of Turner and Scringer (Ref. 12). As shown in Table 7, the theoretical spectrum levels differ significantly from the data of Ref. 12 for low frequencies--almost 5 db in two cases. These deviations may be real, or they may be due largely to data recording and processing differences which can easily introduce variations of several db when the analysis bandwidth is narrow relative to the spectral pattern.

8.4 In some instances, wide discrepancies (up to ~ 8 db) exist between the source level estimates of the present study and calculations using the analytical model of Weston<sup>1</sup>. It has been shown that an inadequate formula for the shock wave impulse was used in Weston's low frequency model, and a less realistic model of the explosion wave form than that of the present study was used at higher frequencies. Limited comparisons with available data indicate the present computational method yields more accurate source level estimates than Weston's analytical model for the 800-ft source.

8.5 The analysis bandwidth used is an important parameter in source level definition. Because of the oscillatory character of explosion spectra, analyses in 1 Hz, 1/3-octave and octave bands may yield far different source level values at the same center frequency.

3.5 Seemingly small variations in burst depth can cause significant variations in narrow-band source level. For example, experimental variation about the 300-ft nominal depth setting for a 1.8-lb SUS may produce as much as 6.3 db source level difference (in the 20 Hz 1/3-octave band) for depth variations of 700 to 900 ft. To avoid this discrepancy, the depth of an explosive source must be accurately determined, and a source level for that depth determined. No simple correction for depth variation can generally be applied to the source level to account for depth variation.

3.7 The effect of the surface reflection, sometimes erroneously included as part of the source level, has been demonstrated in one instance (paragraph 6.10) to produce errors in the source level at different frequencies of as much as +4 and -5 db. Since, for shallow sources, the surface reflection cannot be separated from the direct arrival, direct measurement of these source levels is not possible. (The source level is not necessarily identical to the measured energy spectrum. The latter may include surface reflections and other local effects; the former should not.) Furthermore, no simple correction of a constant number of db will correct the measured spectrum to the correct source level values at all frequencies.

## 9. RECOMMENDATIONS FOR FURTHER WORK

9.1 Further comparisons of the results of this and other calculations with shallow data must be made. These comparisons would evaluate the extrapolation of shock wave parameters at the shallow depths used in this study, as well as determine which approach is best suited for the estimation of shallow charge source levels. Suitable data would consist of broadband (~ 0 - 20K Hz.) pressure-time records, recorded at close range (preferably near 100 yards). These would allow the details of the surface reflection to be analyzed and included in the calculations for comparisons (similar to the procedure of paragraph 7.6). Narrow band recordings make the analysis of time domain characteristics difficult. Such shallow data may be available at other facilities. If not, suitable field work should be planned to gather the needed data.

9.2 In order to evaluate the effect of different analysis techniques, several samples of data should be processed through different analysis systems, digital, analog and hybrid digital-analog, and the results compared. Since our analysis capability is digital, we would be eager to exchange data with facilities using other techniques.

9.3 In the absence of the above additional comparisons, we tentatively recommend that shallow explosive source levels, where the surface reflection cannot be separated out, be determined by the method of the present study. As direct measurement of these source levels is impossible, we feel that the promising comparisons of section 7 indicate that this method yields the most accurate available estimates for such shallow sources. Close-in pressure measurements are still needed in particular applications so that the actual source depth may be determined, and the error introduced by source depth variation eliminated.

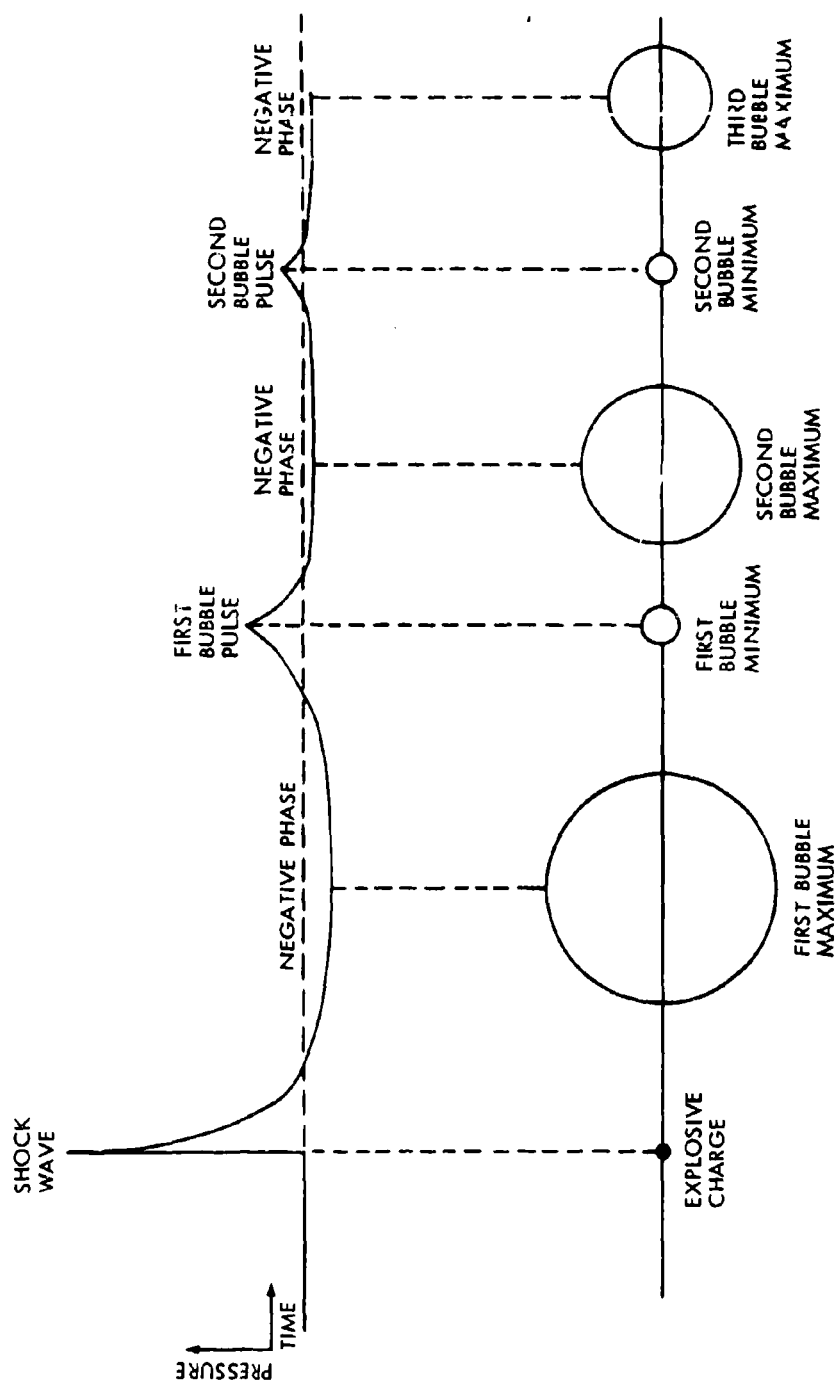
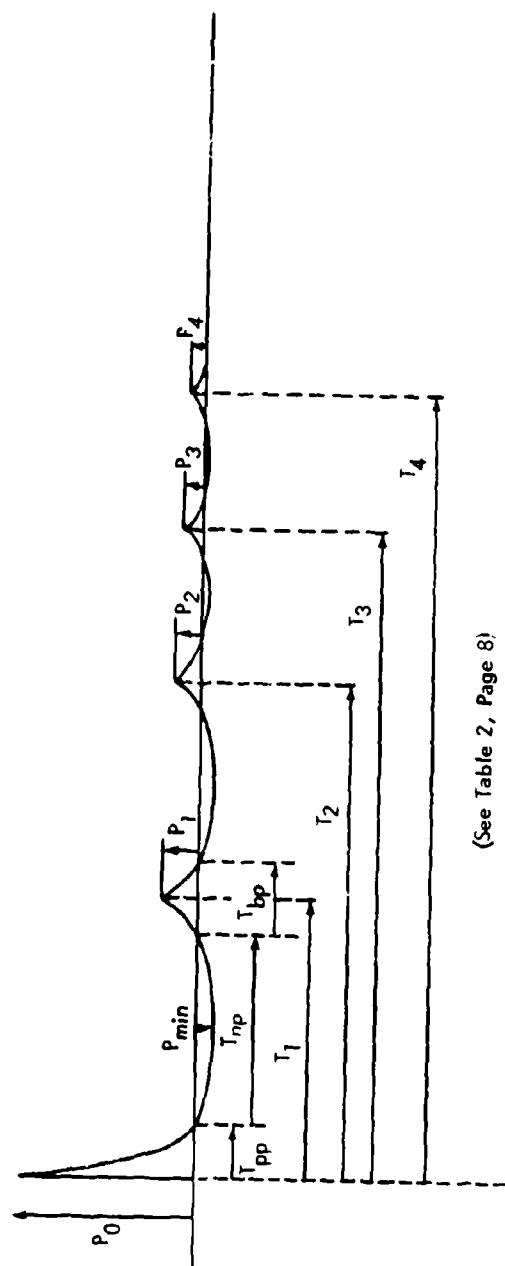


FIG. 1. EXPLOSION BUBBLE AND PRESSURE-TIME HISTORY



(See Table 2, Page 8)

FIG. 2. PARAMETERS OF THE PRESSURE-TIME HISTORY



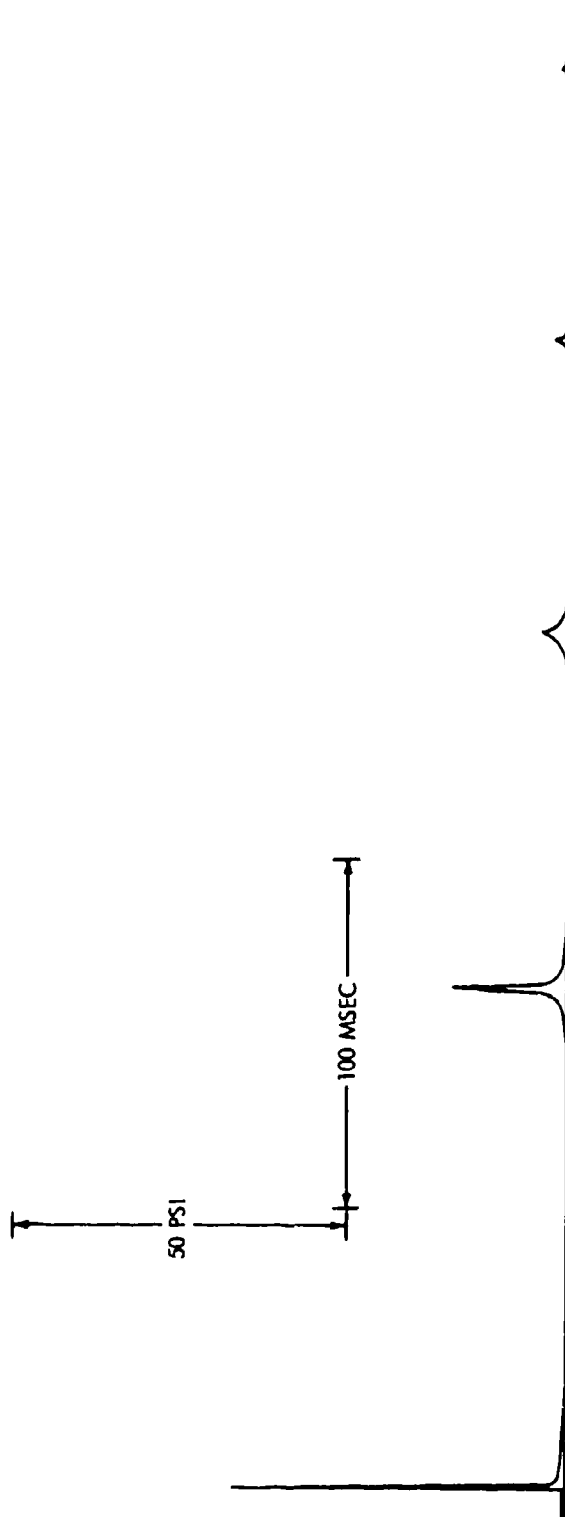


FIG. 3. IDEALIZED PRESSURE-TIME CURVE-3 POUNDS OF TNT AT 60 FOOT BURST DEPTH - 100 YARD RANGE

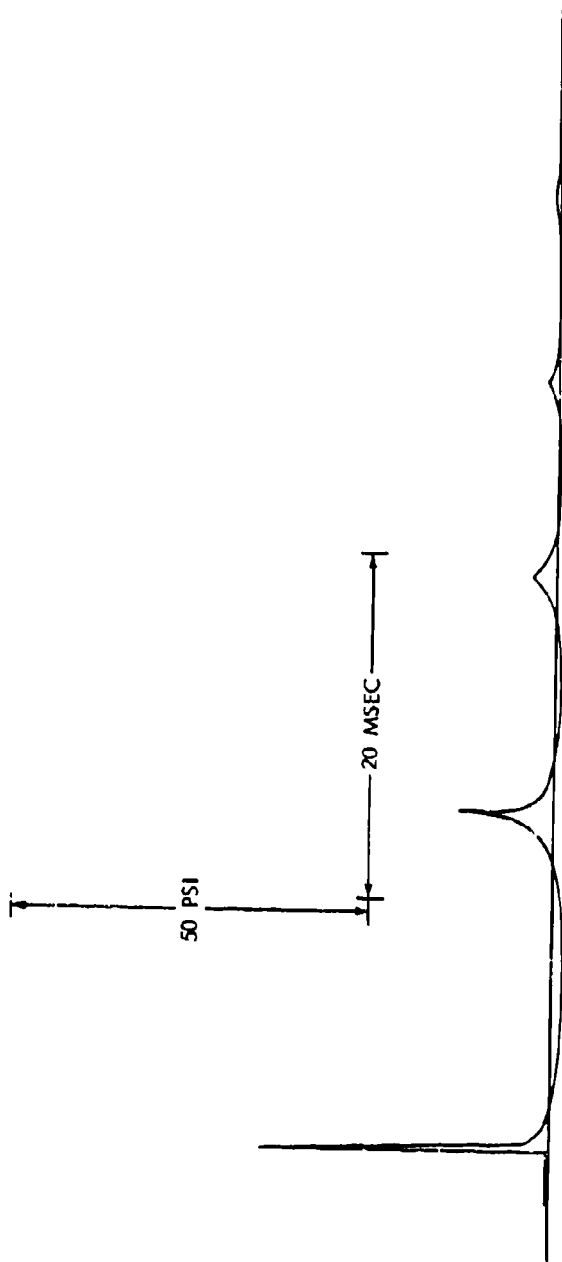


FIG. 4. IDEALIZED PRESSURE-TIME CURVE-1.8 POUNDS OF TNT AT 800 FOOT BURST DEPTH — 100 YARD RANGE

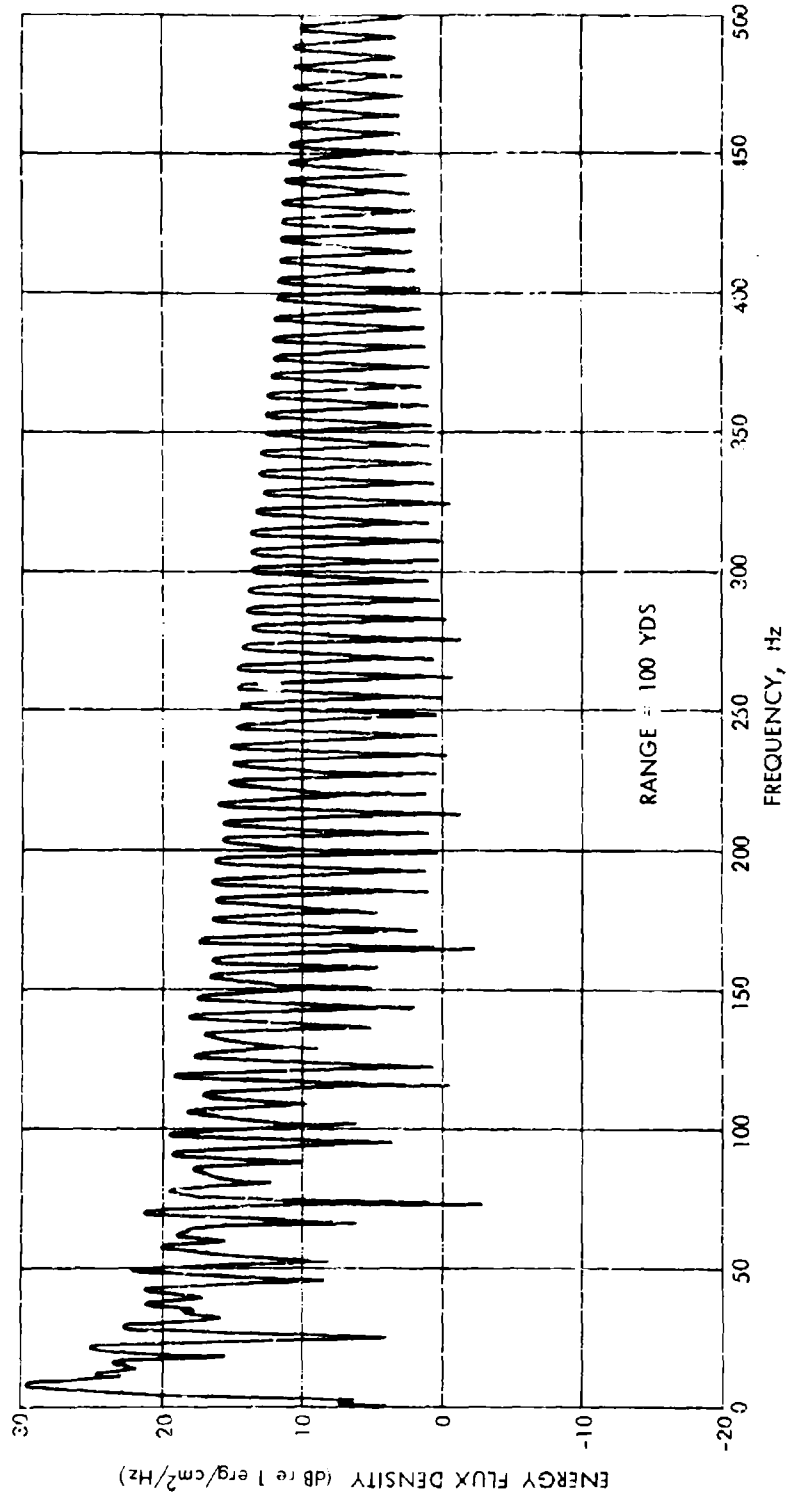


FIG. 5. IDEALIZED SOURCE LEVEL-3 LB OF TNT AT 60 FT DEPTH

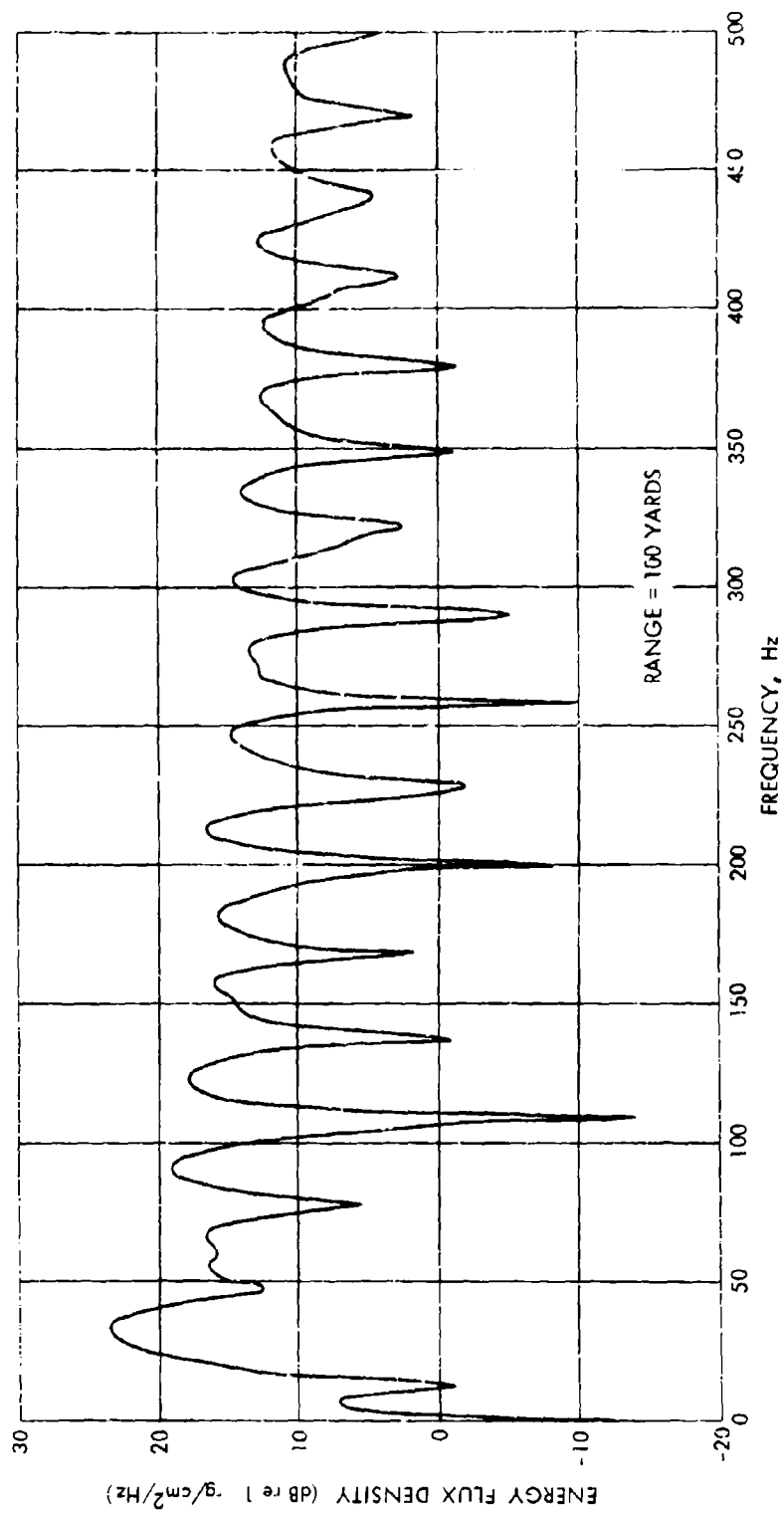


FIG. 6. IDEALIZED SOURCE LEVEL-3 LB OF TNT AT 500 FOOT DEPTH

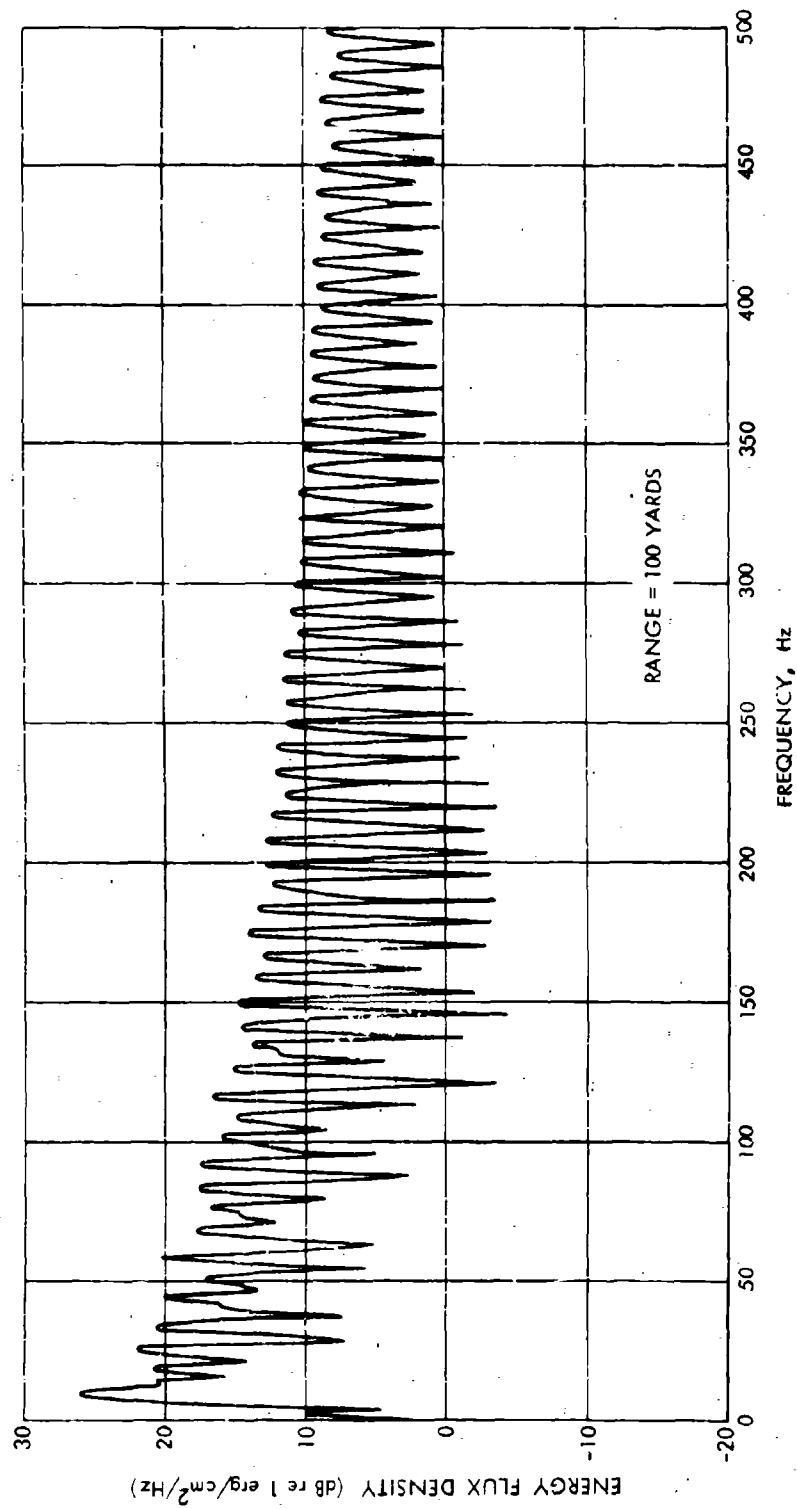


FIG. 7. IDEALIZED SOURCE LEVEL-1.8 LB OF TNT AT 60 FOOT DEPTH

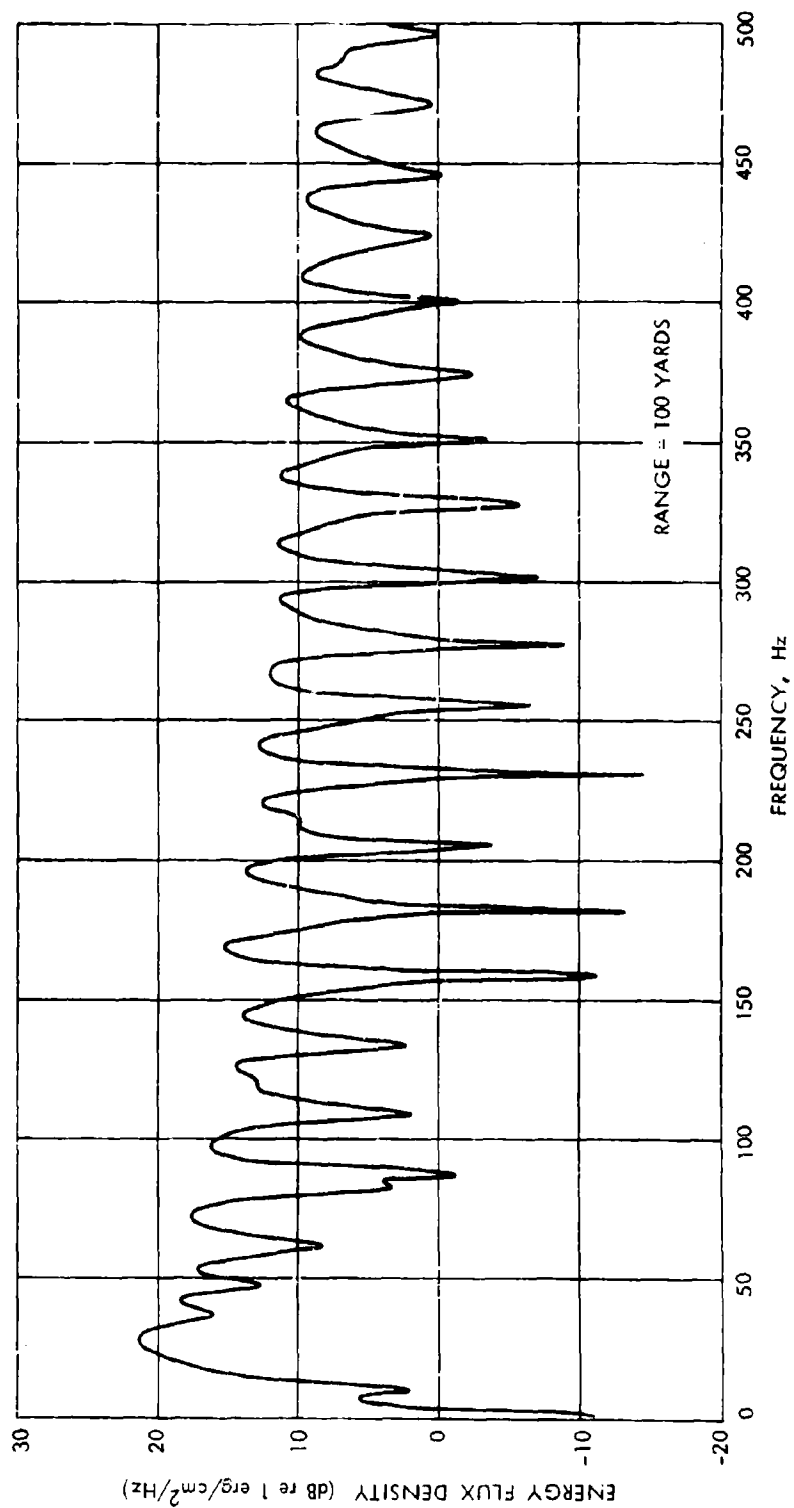


FIG. 8. IDEALIZED SOURCE LEVEL-1.8 LB OF TNT AT 300 FOOT DEPTH

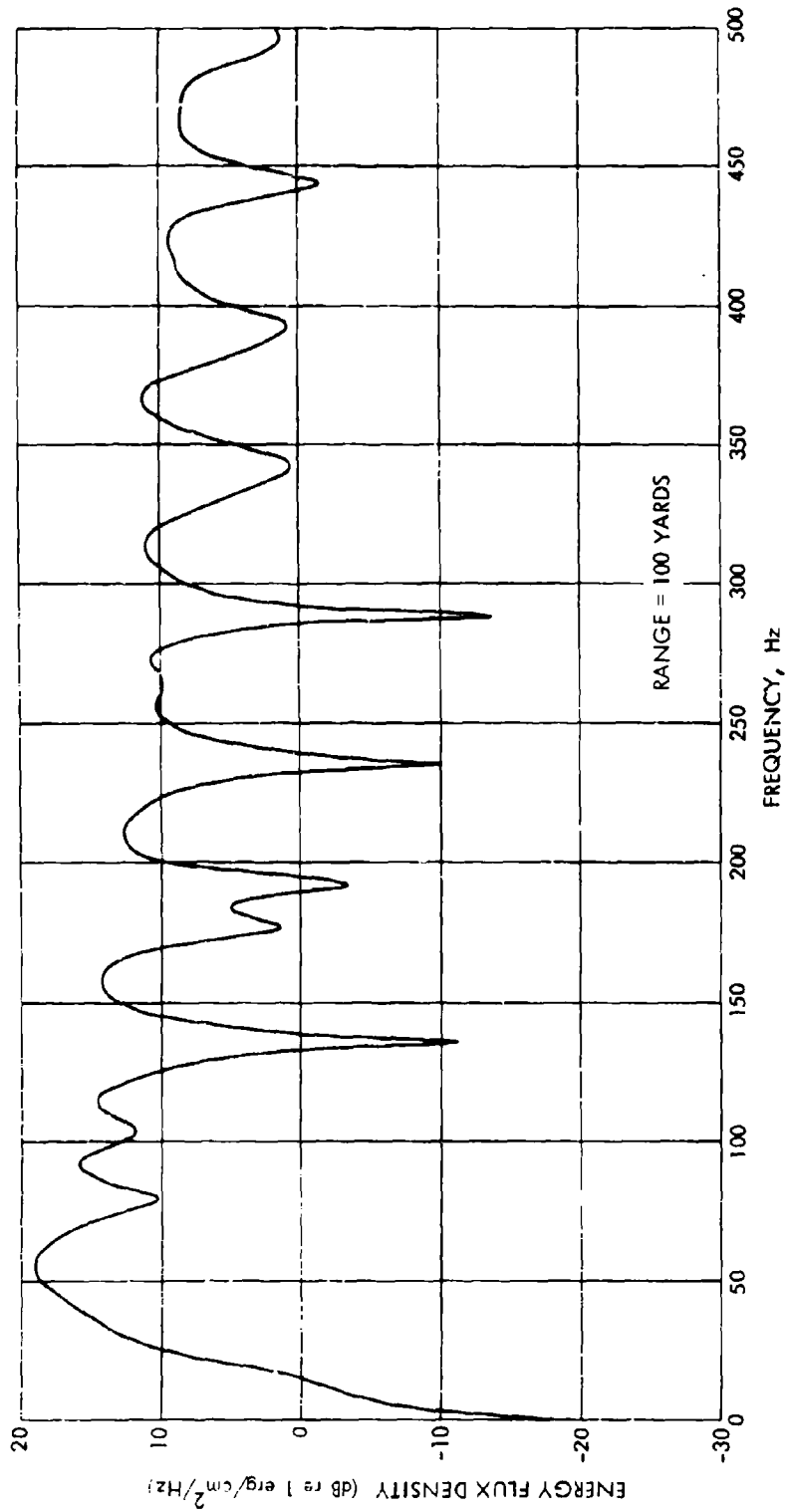


FIG. 9. IDEALIZED SOURCE LEVEL-1.8 LB OF TNT AT 800 FOOT DEPTH

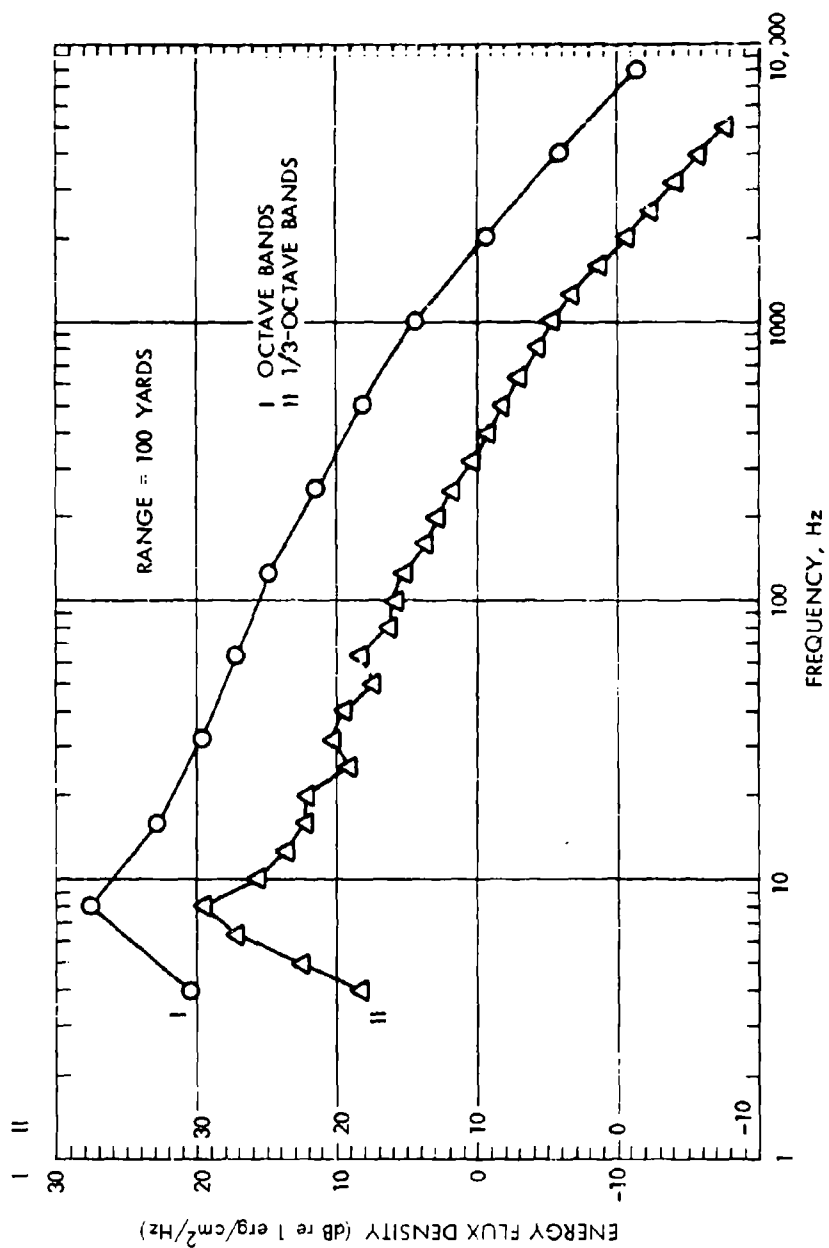


FIG. 10. OCTAVE AND 1/3-OCTAVE BAND SOURCE LEVELS - 3 POUNDS OF TNT AT 60 FOOT BURST DEPTH



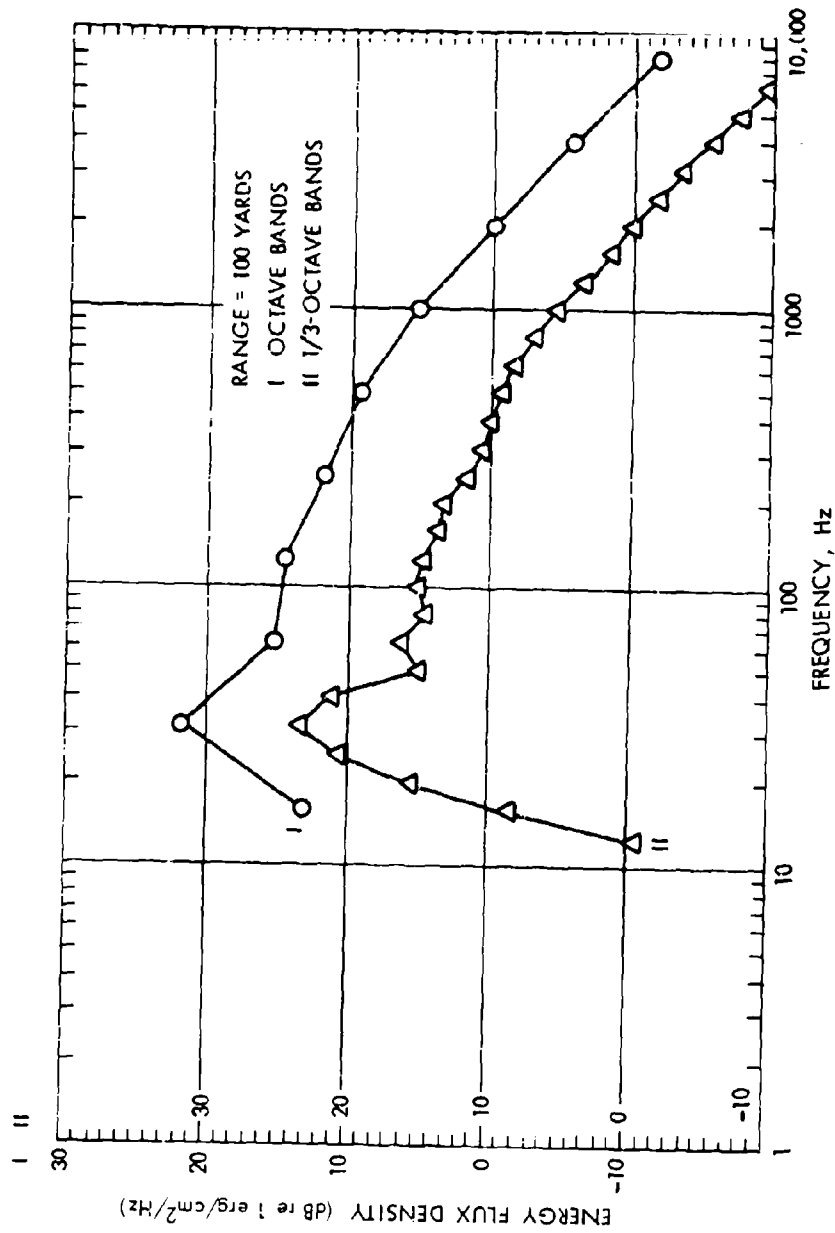


FIG. 11. OCTAVE AND 1/3-OCTAVE BAND SOURCE LEVELS - 3 POUNDS OF TNT AT 500 FOOT BURST DEPTH

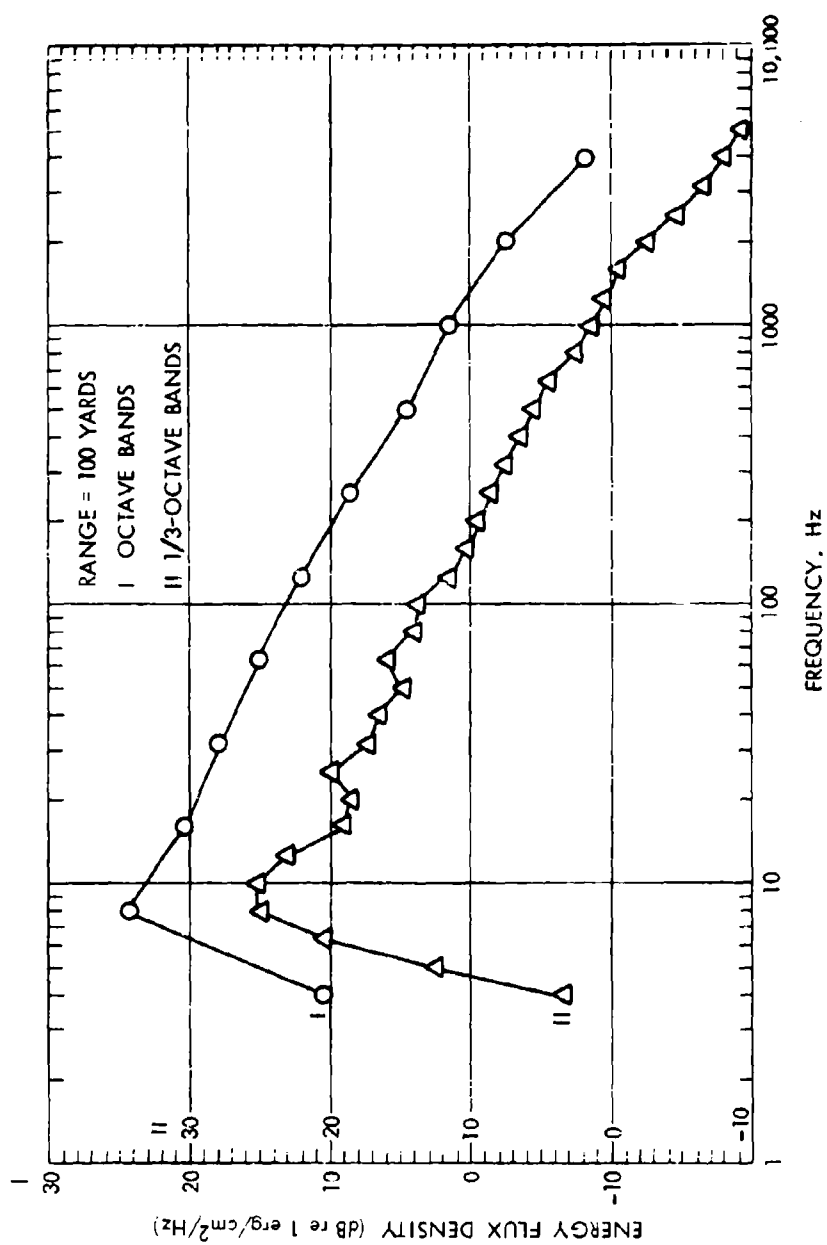


FIG. 12. OCTAVE AND 1/3-OCTAVE BAND SOURCE LEVELS ~ 1.8 POUNDS OF TNT AT 60 FOOT BURST DEPTH

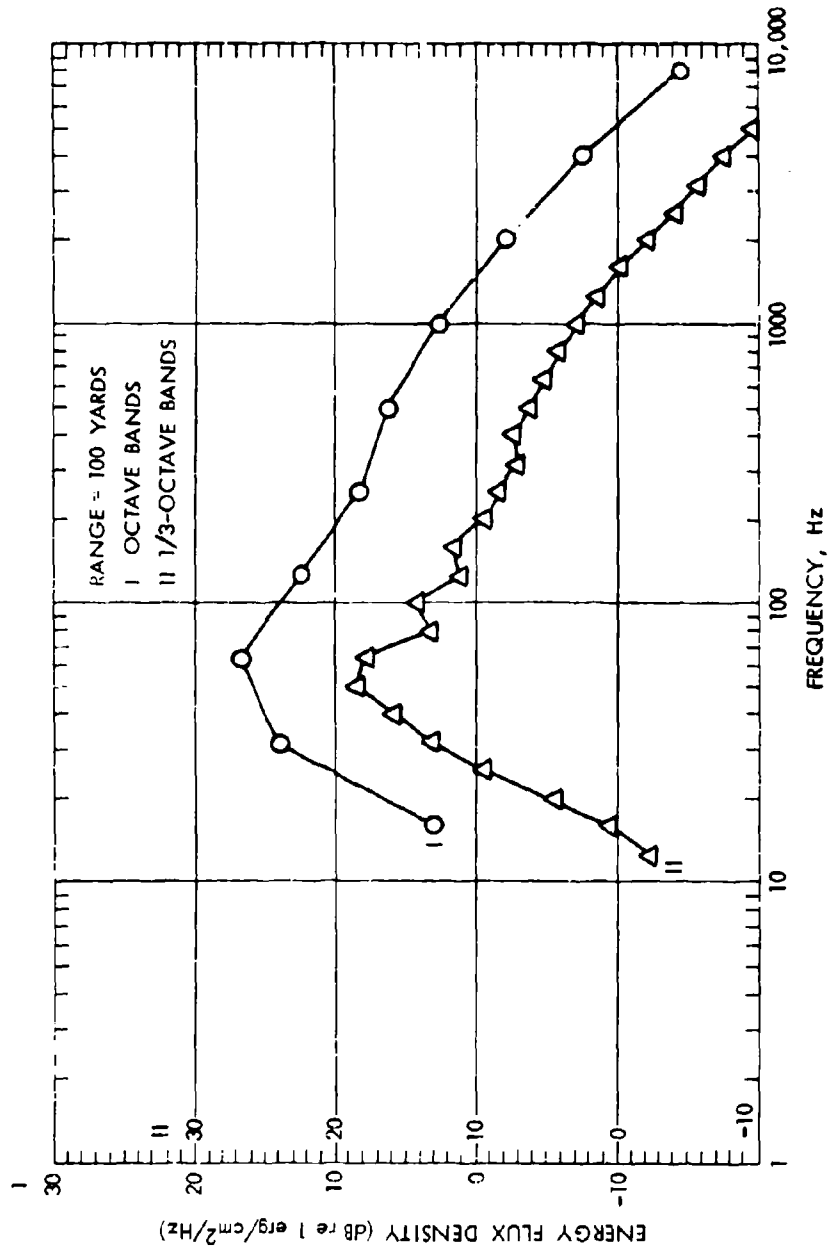


FIG. 13. OCTAVE AND 1/3-OCTAVE BAND SOURCE LEVELS ~ 1.8 POUNDS OF TNT AT 300 FOOT BURST DEPTH

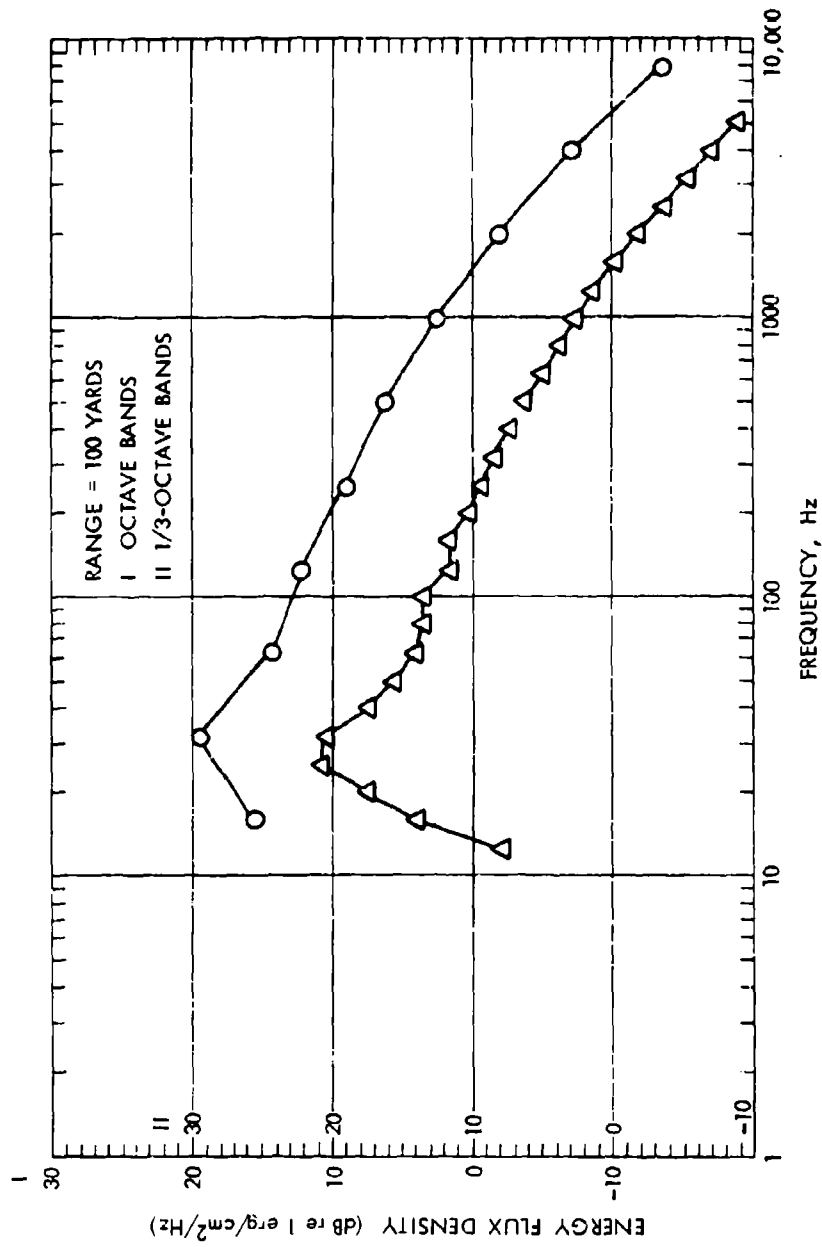


FIG. 14. OCTAVE AND 1/3-OCTAVE BAND SOURCE LEVELS - 1.8 POUNDS OF TNT AT 800 FEET BURST DEPTH

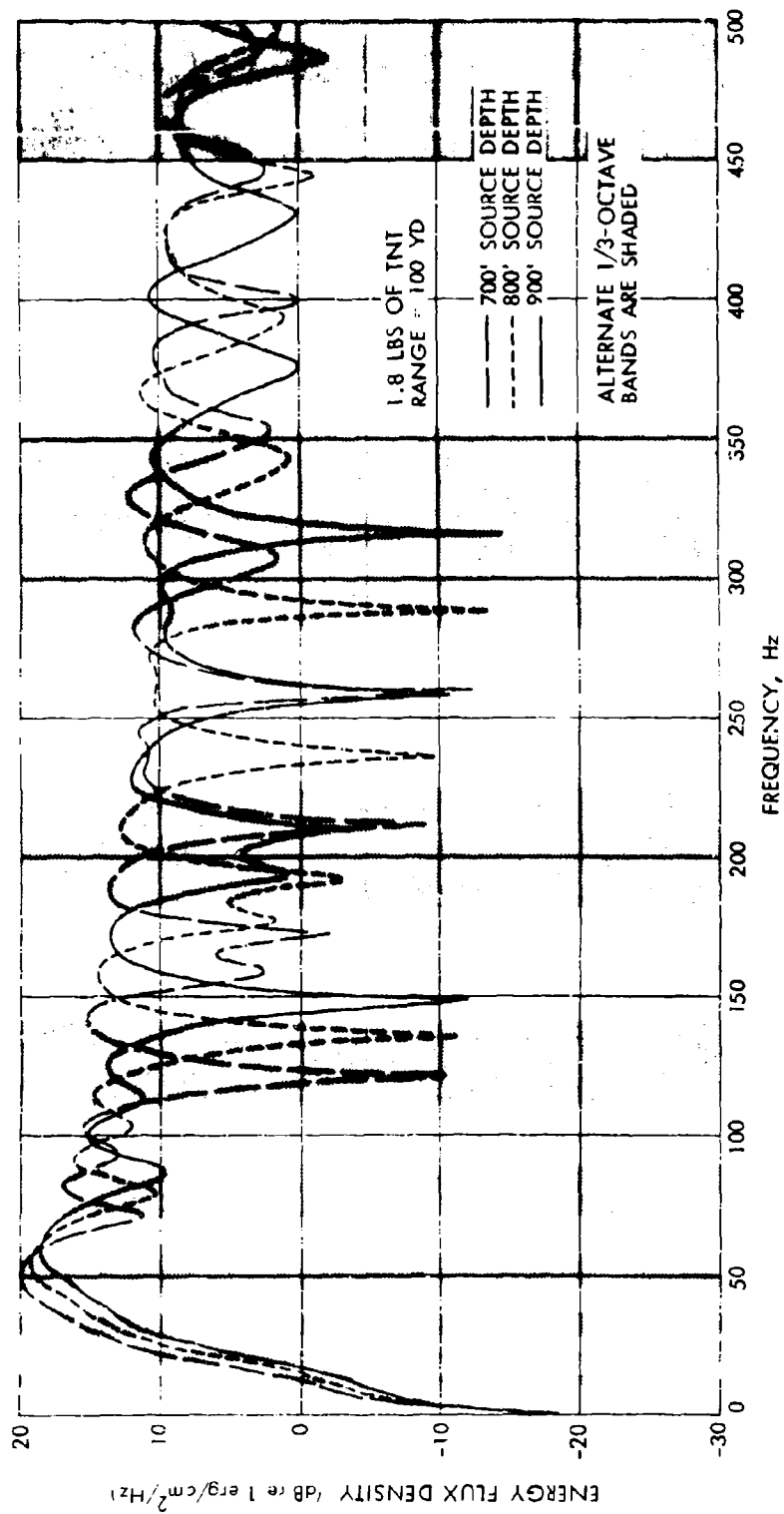


FIG. 15. EFFECT OF SMALL VARIATIONS IN BURST DEPTH

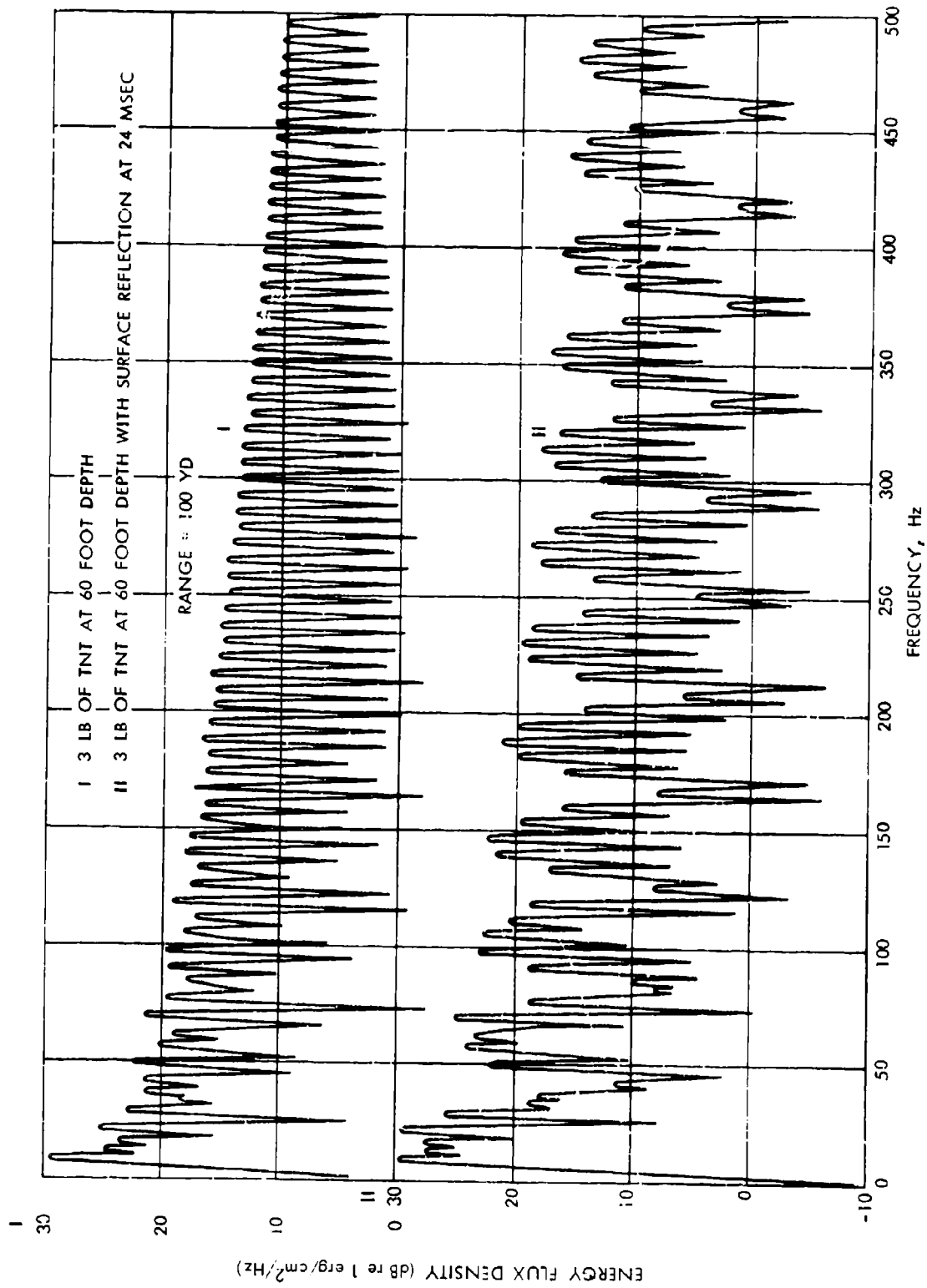


FIG. 16. DETAILED ENERGY SPECTRA SHOWING EFFECT OF SURFACE REFLECTION

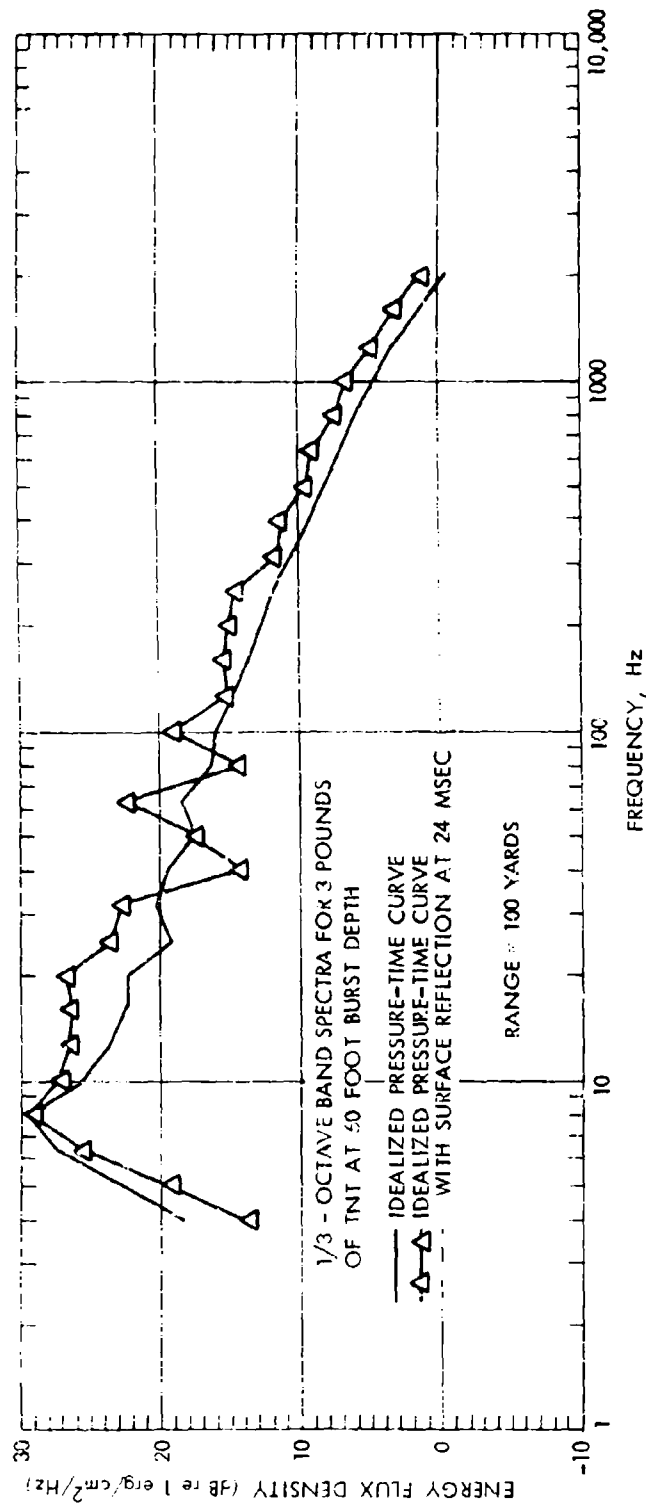


FIG. 17. ERROR IN SOURCE LEVEL DUE TO INCLUSION OF SURFACE REFLECTION

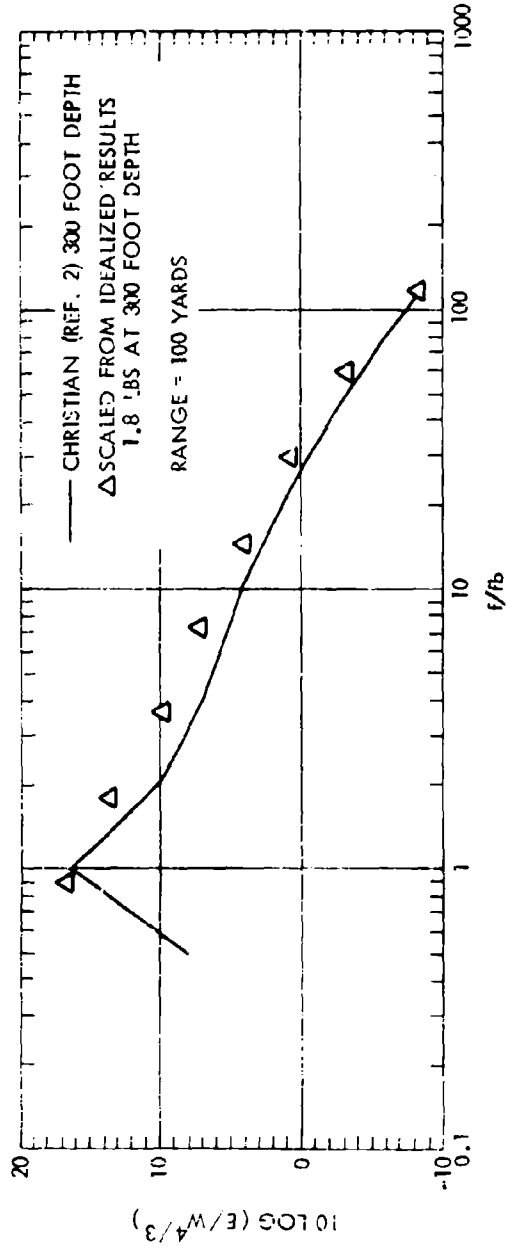


FIG. 18. OCTAVE BAND COMPARISON WITH CHRISTIAN'S SOURCE LEVELS — 300 FOOT DEPTH



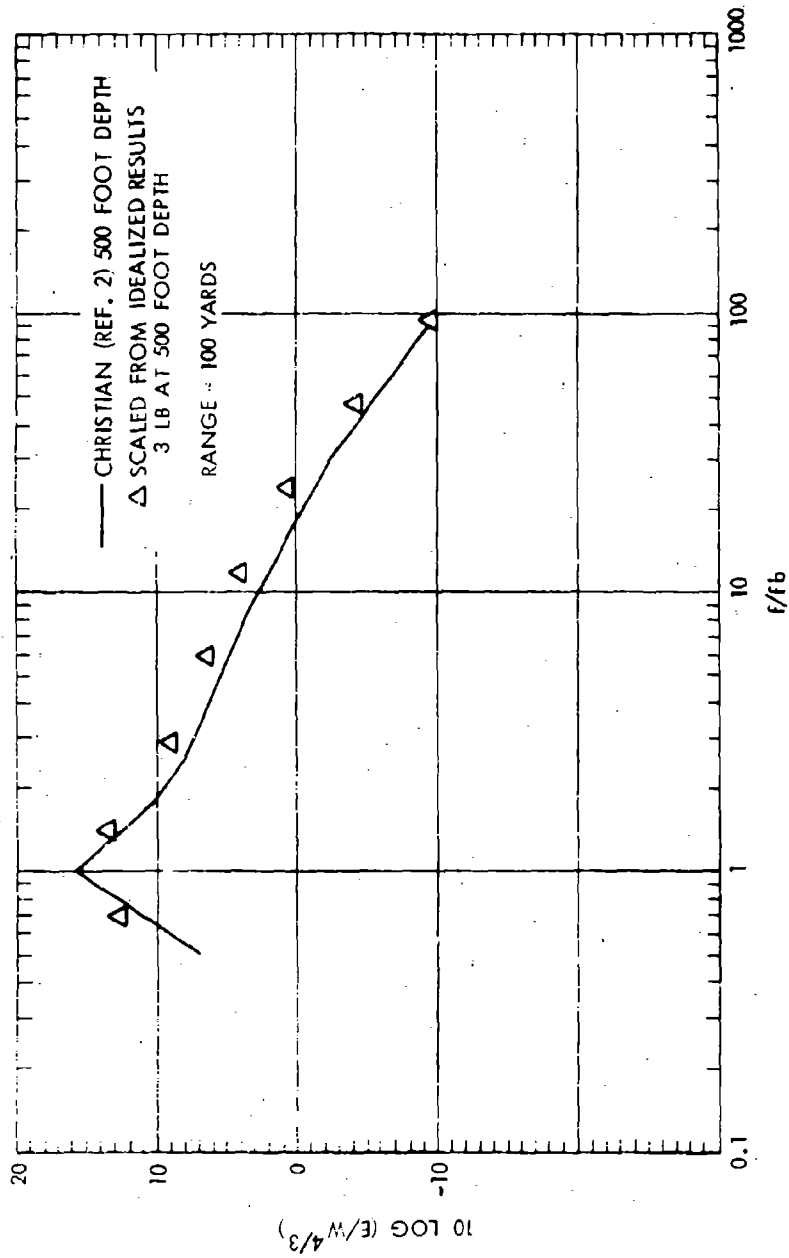


FIG. 19. OCTAVE BAND COMPARISON WITH CHRISTIAN'S SOURCE LEVELS — 500 FOOT DEPTH

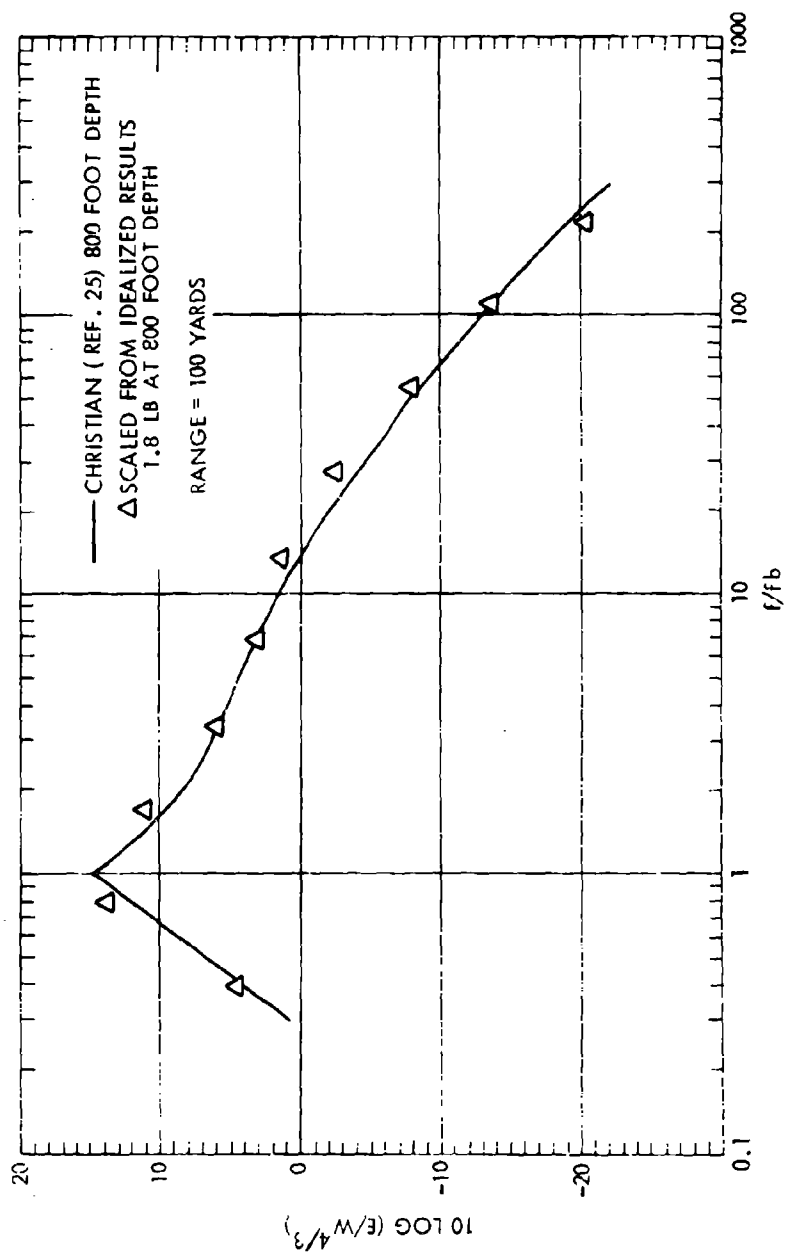


FIG. 20. OCTAVE BAND COMPARISON WITH CHRISTIAN'S SOURCE LEVELS — 800 FOOT DEPTH

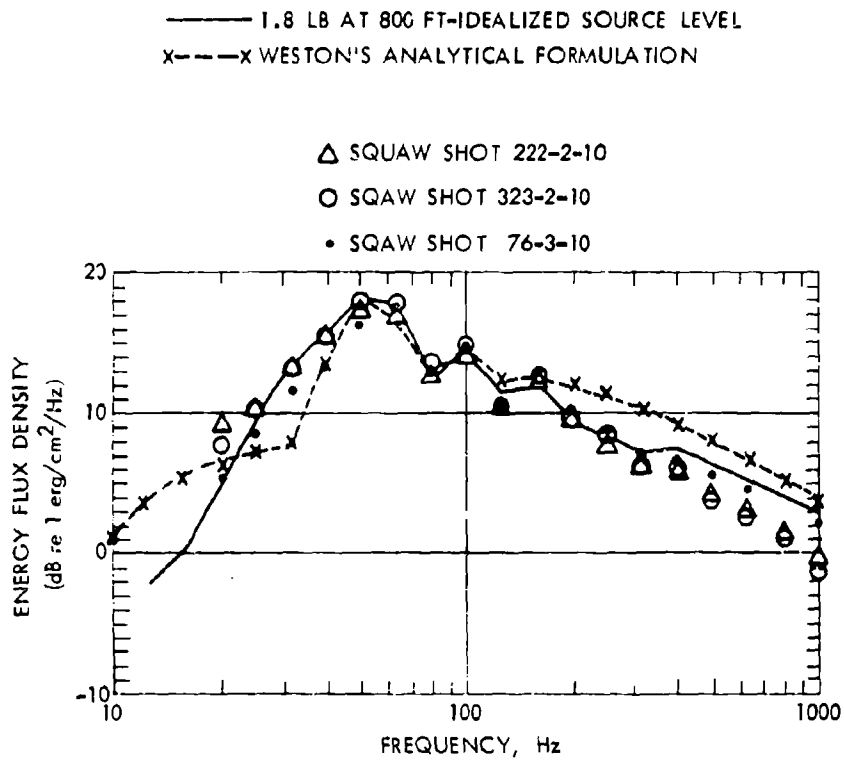


FIG. 21. ONE-THIRD OCTAVE BAND SOURCE LEVEL COMPARISONS - 800 FOOT BURST DEPTH

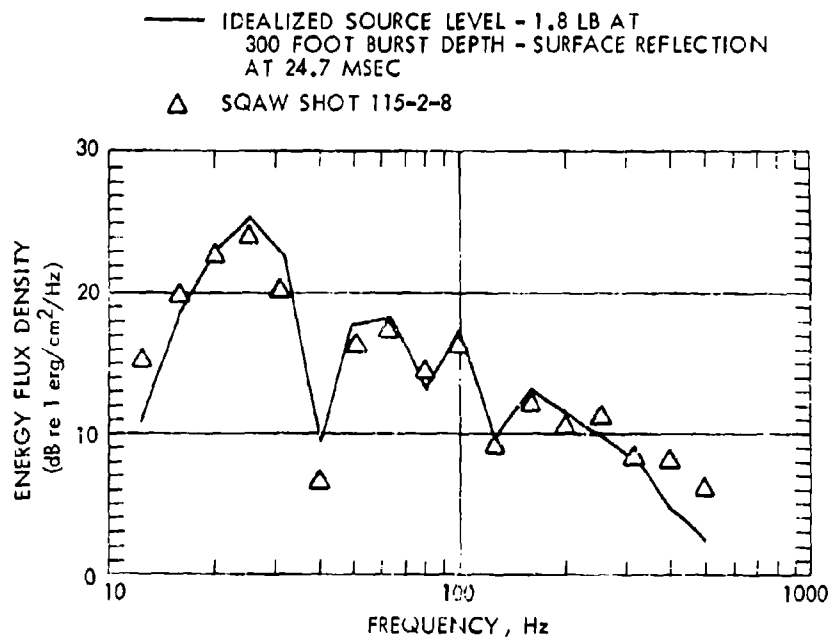


FIG. 22. ONE-THIRD OCTAVE BAND SOURCE LEVEL COMPARISON - 300 FOOT BURST DEPTH

IMPULSE FROM REF. 16 =  $I_S$   
IMPULSE FROM REF. 18 =  $I_W$

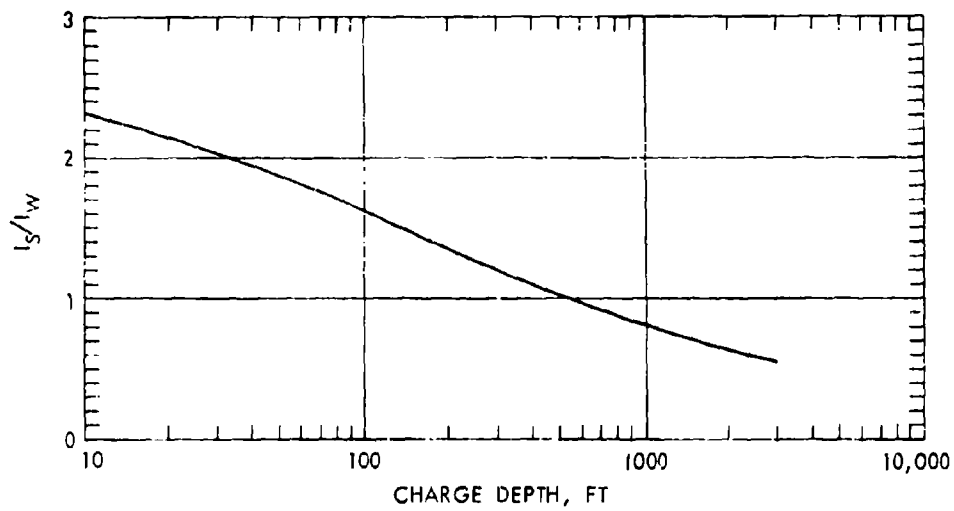


FIG. 23. IMPULSE RATIO

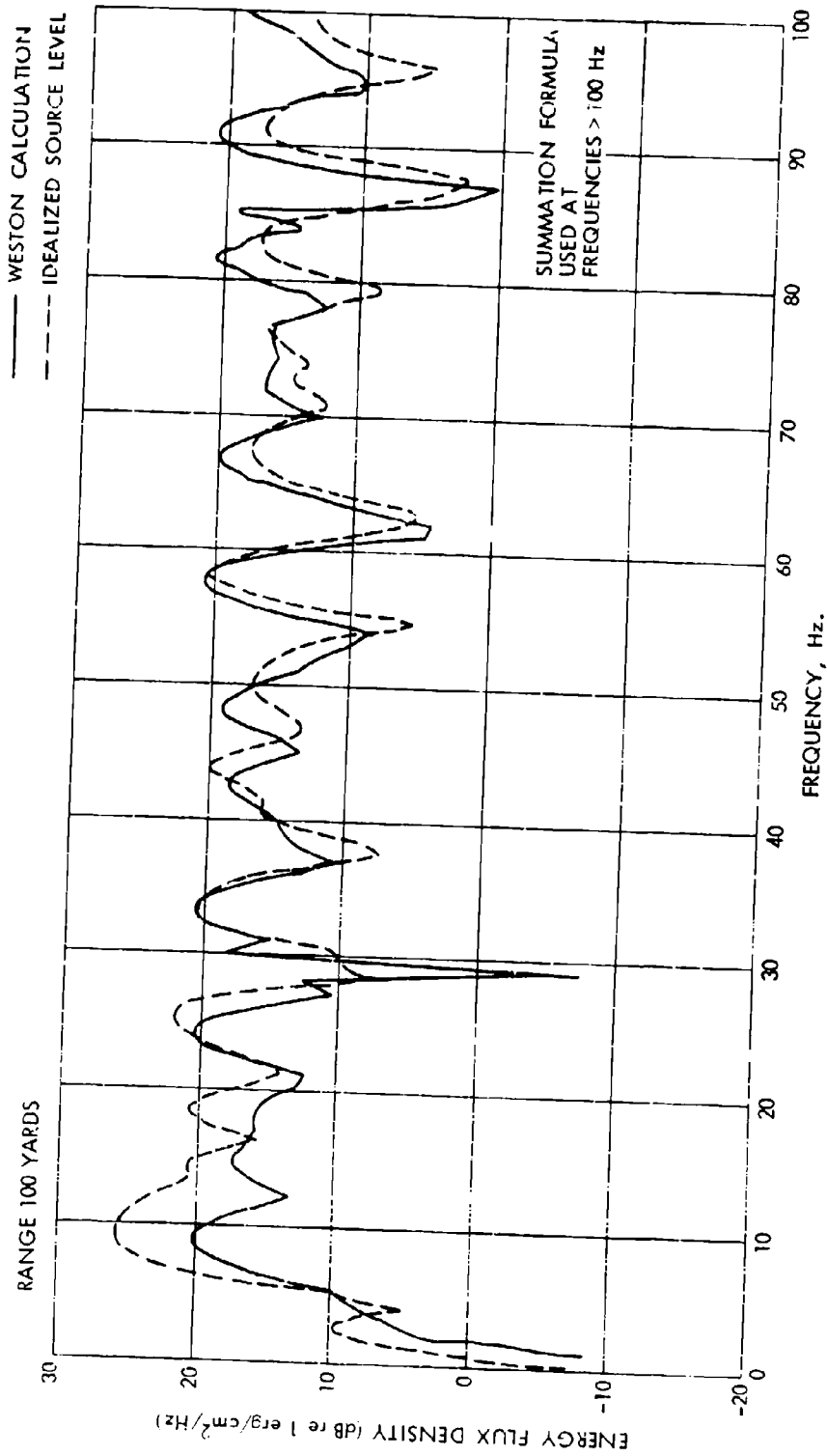


FIG. 24. COMPARISON WITH WESTON'S ANALYTICAL FORMULATION — 1.8 LB AT 60 FT

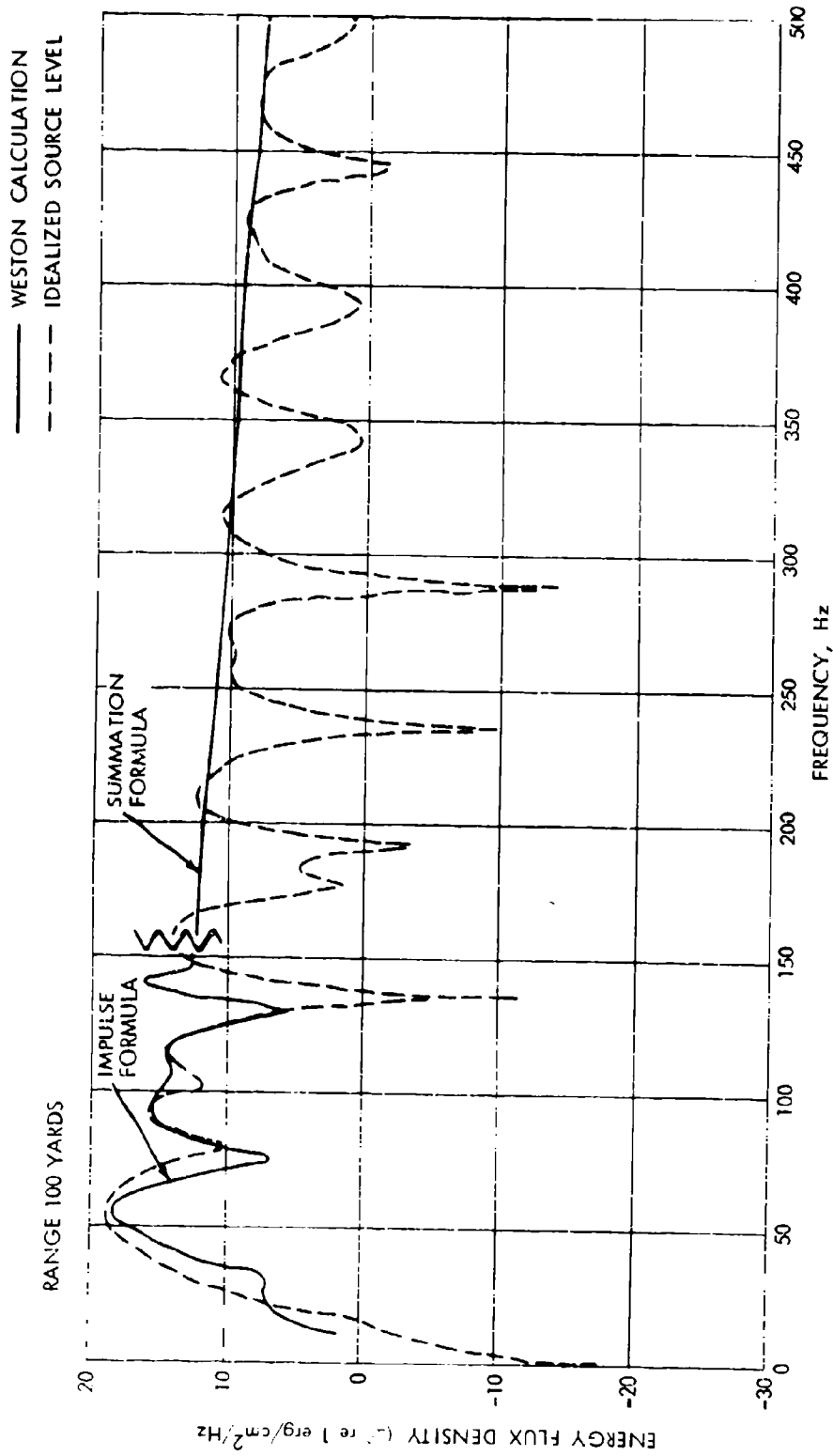


FIG. 25. COMPARISON WITH WESTON'S ANALYTICAL FORMULATION - 1.8 LB AT 800 FT

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